Discrete Structures (CSCI 246)

in-class activity

Names: ____

We have seen that we can find a function to represent an algorithm's runtime as the number of primitive operations it performs on an input n. In today's activity, your group will try to come up with your own *precise, mathematical* way to compare two such functions. There is no "correct" answer here! In fact, it would be amazing if all groups came up with something different.

You must come up with something by the end of class, so please work through this worksheet at a pace that allows you to finish on time.

- First, designate a scribe for your group who will fill in this worksheet to turn in. Write their name here: ______________________________. Please only turn in that single worksheet for your group.
- There could be many ways to compare functions. To best match what is currently done in the computer science field, your definition should try to do the following three things.
  1. We want to compare based on whether a function f grows no faster than a function g, or roughly, whether $f \leq g$. (Note that this says less than *or equal*!)
  2. We want to capture the behavior of the functions for large n; we don't care about small n.
  3. We want a generic view of the functions' behavior, not a specific view.
- Note: just like $\leq$ is a relation on the set of integers, your way of comparing functions is a relation on the set of all functions $f : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$. Let's call your group's relation $\leq_x$, where x is your group's number.
- In order to write a definition, we need to be sure that we agree on what our definition is trying to capture. For the functions below, draw an arrow from f to g if $f \leq_x g$. (Things to consider: are there self-loops? Are there any cycles?) All of the functions are graphed together on the next page for reference if you need.

$$f_2(n) = 2^n$$

$$f_1(n) = 3n^2 - 100n + 2$$

$$f_6(n) = 10n + 5000$$

$$f_5(n) = \begin{cases} 6n^2 + 2n + 3 & \text{if } n \text{ is odd,} \\ 6 & \text{otherwise} \end{cases}$$

$$f_3(n) = n^2$$

$$f_4(n) = \log_2 10n$$

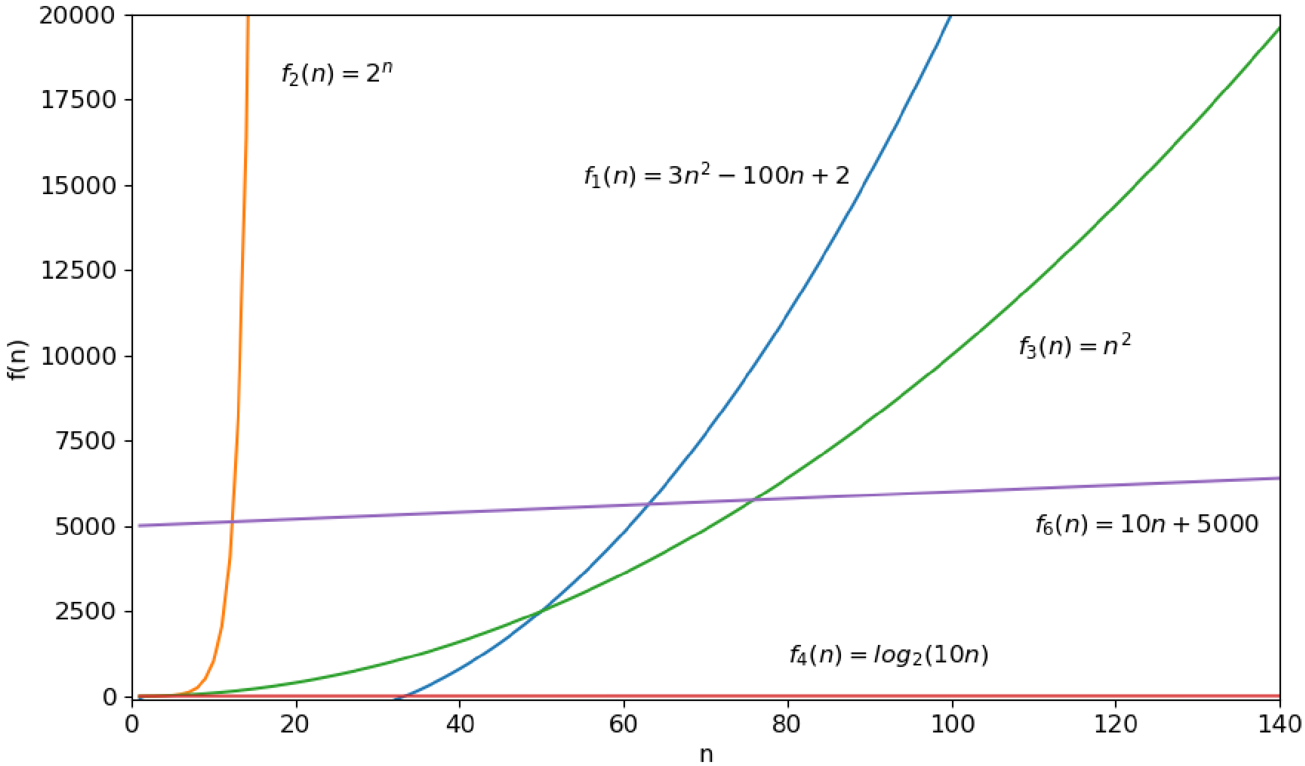

Figure 1: Functions from Page 1. Note that f_5 is excluded, because it jumps from 6 to $6n^2 + 2n + 3$ and would clutter the graph, but there is a graph with f_5 on the final page.

- Now that you have agreed about which functions are $\leq_x$ other functions, try to describe how you know *precisely*, but for a generic function. Ideally, you should be able to fit your definition into a form like this:

  Let $f : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ and $g : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$ be functions. $f \leq_x g$ if...[some proposition that you come up with is true. This proposition will probably have some $\exists$s and $\forall$s in it, and use the fact that you can get $f(x)$ and $g(x)$ for $x \geq 0$.].

- If you finish, try to apply your definition to $f = f_5$ and $g = f_3$ from above.

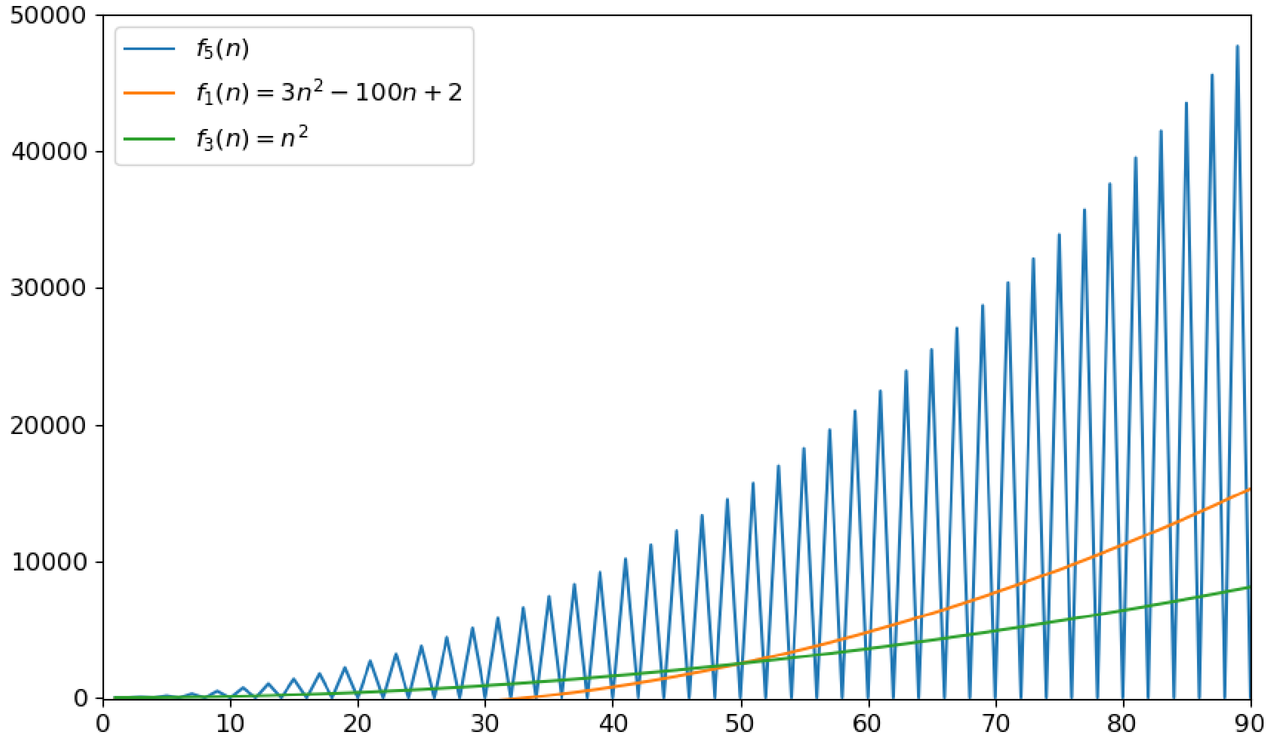

Figure 2: Graph of $f_5(n) = \begin{cases} 6n^2 + 2n + 3 & \text{if } n \text{ is odd,} \\ 6 & \text{otherwise} \end{cases}$, along with two other quadratic functions.