

2. Prove

$$\underbrace{P(A) \cup P(B)}_G \subseteq \underbrace{P(A \cup B)}_H ?$$

Prove an equivalent claim,

when  $x \in G$ ,  $x \in H$

if  $x \in G$ , then  $x \in H$ .

Proof Assume  $x \in G$ . WTS  $x \in H$ .

→ ∴

$x \in H$ .

$$q \Rightarrow p$$

$p$	$q$	$\neg q$	$\neg p$	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$	$\neg p \Rightarrow \neg q$
T	T	F	F	T	T	T
T	F	T	F	F	F	F
F	T	F	T	T	T	T
F	F	T	T	T	T	T

$$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$$

"logically equivalent to"

$\neg q \Rightarrow \neg p$  is the contrapositive of  $p \Rightarrow q$

is  $\neg p \Rightarrow \neg q \equiv p \Rightarrow q$ ? No.

What is the converse of  $p \Rightarrow q$ ?

$$q \Rightarrow p$$

$$q \Rightarrow p \equiv p \Rightarrow q$$

$q$	$p$	$q \Rightarrow p$
T	T	T
T	F	F
F	T	T
F	F	T

$$p \Rightarrow q$$

Claim If  $n^2$  is even, then  $n$  is even.

① For contradiction, suppose  $\neg(p \Rightarrow q)$

②  $\neg(p \Rightarrow q) \equiv p \wedge \neg q$

③ proof of  $\neg q \Rightarrow \neg p$

④ Note that we have  $\neg p \wedge p$

⑤  $\neg(p \Rightarrow q)$  is false, so  $p \Rightarrow q$  is T

Proof For contradiction, suppose the claim is false. That is, suppose that  $n^2$  is even but  $n$  is odd.

•  $n = 2k + 1$  for  $k \in \mathbb{Z}$

$$n^2 = (2k + 1)^2$$

$$n^2 = 4k^2 + 4k + 1$$

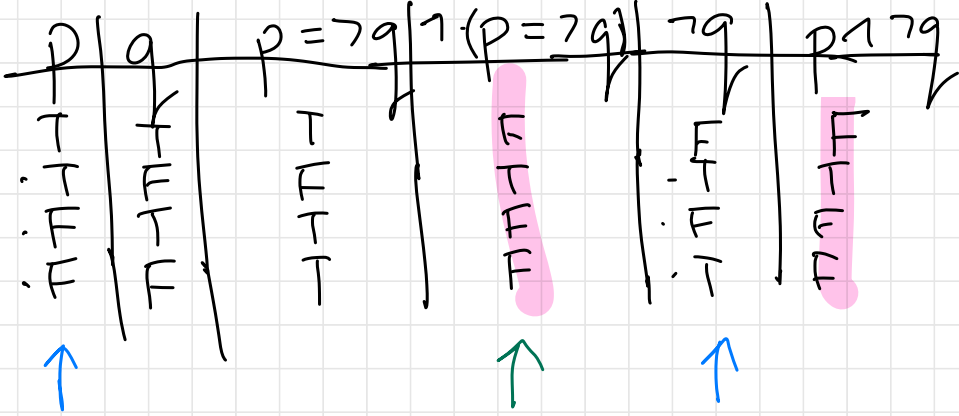
$$n^2 = 2(2k^2 + 2k) + 1$$

$$n^2 = 2c + 1 \text{ for } c \in \mathbb{Z}$$

•  $n^2$  is odd  $\neg q$

This contradicts that  $n^2$  is even.

So our initial assumption that the claim is false is false, so the claim is true.



Claim If  $n^2$  is even, then  $n$  is even.

Proof We give a proof by contrapositive.

That is, we prove that

If  $n$  is odd, then  $n^2$  is odd.

Assume  $n$  is odd. WTS  $n^2$  is odd.

$$n = 2k + 1 \text{ for } k \in \mathbb{Z}$$

$n$  is odd  
def. of odd

$$n^2 = (2k + 1)^2$$

$$n^2 = 4k^2 + 4k + 1$$

$$n^2 = 2(2k^2 + 2k) + 1$$

$$n^2 = 2c + 1 \text{ for } c \in \mathbb{Z}$$

$n^2$  is odd

□