2. Prove

$$\underbrace{\mathcal{P}(A) \cup \mathcal{P}(B)}_{G} \subseteq \underbrace{\mathcal{P}(A \cup B)}_{H} \; ?$$

Prove an equivalent claim,

when $x \in G$, $x \in H$

if $x \in G$, then $x \in H$.

Proof Assume $x \in G$. WTS $x \in H$.

$\Rightarrow$ $\vdots$

$x \in H$.

$q \Rightarrow p$

$\neg q \Rightarrow \neg p$

p	q	$\neg q$	$\neg p$	$p \Rightarrow q$	$\neg q \Rightarrow \neg p$	$\neg p \Rightarrow \neg q$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	F
F	F	T	T	T	T	T

$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

"logically equivalent to"

$\neg q \Rightarrow \neg p$ is the <u>contrapositive</u> of $p \Rightarrow q$

is $\neg p \Rightarrow \neg q \equiv p \Rightarrow q$? No.

what is the converse of $p \Rightarrow q$?

$q \Rightarrow p$

Is $q \Rightarrow p \equiv p \Rightarrow q$

q	p	$q \Rightarrow p$
T	T	T
F	T	T
T	F	F
F	F	T

$p \Rightarrow q$

Claim: If $\overbrace{n^2 \text{ is even}}^{p}$, then $\overbrace{n \text{ is even}}^{q}$.

(1) For contradiction, suppose $\neg(p \Rightarrow q)$

(2) $\neg(p \Rightarrow q) \equiv p \wedge \neg q$

(3) proof of $\neg q \Rightarrow \neg p$

(4) Note that we have $\neg p \wedge p$

(5) $\neg(p \Rightarrow q)$ is false, so $p \Rightarrow q$ is T

Proof: For contradiction, suppose the claim is false. (1) That is, suppose that $\underbrace{n^2 \text{ is even}}_{p}$ but $\underbrace{n \text{ is odd}}_{\neg q}$. (2)

$$\left.\begin{aligned}
&\bullet\ n = 2k+1 \text{ for } k \in \mathbb{Z} \\
&n^2 = (2k+1)^2 \\
&n^2 = 4k^2 + 4k + 1 \\
&n^2 = 2(2k^2+2k)+1 \\
&n^2 = 2c+1 \text{ for } c \in \mathbb{Z} \\
&\bullet\ n^2 \text{ is odd } \neg p
\end{aligned}\right\}$$

assume $\neg q$

(3)

derive $\neg p$

This contradicts that $\overset{p}{n^2 \text{ is even}}$. (4)

So our initial assumption that the claim is false is false, so the claim is true. (5)

p	q	$p \Rightarrow \neg q$	$\neg(p \Rightarrow \neg q)$	$\neg q$	$p \wedge \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

<u>Claim</u> If n^2 is even, then n is even.

<u>Proof</u> We give a proof by contrapositive.

That is, we prove that

If n is odd, then n^2 is odd.

Assume n is odd. WTS n^2 is odd.

$n = 2k+1$ for $k \in \mathbb{Z}$ n is odd
def. of odd

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

$n^2 = 2c + 1$ for $c \in \mathbb{Z}$

n^2 is odd

$\square$