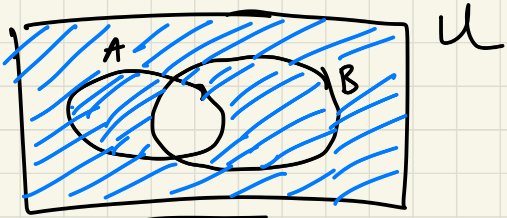
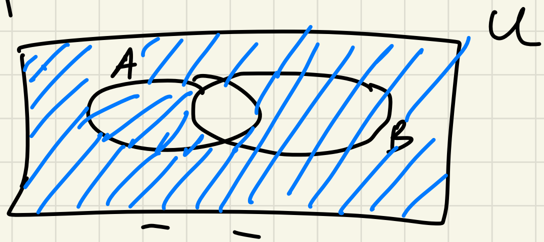
Thm (De Morgan's) $\overline{A \cap B} = (\bar{A} \cup \bar{B})$

The complement of the intersection is the union of the complements.



$\overline{A \cap B}$



$\bar{A} \cup \bar{B}$

Pf We will prove that

$$\left.\begin{array}{l} \overline{A \cap B} \subseteq (\bar{A} \cup \bar{B}) \text{ and} \\ (\bar{A} \cup \bar{B}) \subseteq \overline{A \cap B} \end{array}\right\} \Rightarrow \begin{array}{l} \overline{A \cap B} = (\bar{A} \cup \bar{B}) \\ x \in \text{ iff } x \in \end{array}$$

$\overline{A \cap B} \subseteq (\bar{A} \cup \bar{B})$: we will show that if $x \in \overline{A \cap B}$, then $x \in (\bar{A} \cup \bar{B})$.

Suppose $x \in \overline{A \cap B}$. WTS $x \in \bar{A} \cup \bar{B}$.

$x \notin A \cap B$	by def. of $\overline{}$
x not in both A and B	by def. of $\notin$ and $\cap$
$x \notin A$ or $x \notin B$	by reasoning formalized later
$x \in \bar{A}$ or $x \in \bar{B}$	by def. of $\overline{}$
$x \in \bar{A} \cup \bar{B}$	by def. of $\cup$

Now suppose $x \in \bar{A} \cup \bar{B}$. WTS $x \in \overline{A \cap B}$.
$x \in \bar{A}$ or $x \in \bar{B}$ by definition of $\cup$. So we will prove by cases.

Case 1: $x \in \bar{A}$. WTS $x \in \overline{A \cap B}$.

$x \notin A$	by def. of $\overline{}$
$x \notin A \cap B$	since $A \cap B \subseteq A$
$x \in \overline{A \cap B}$	by def. of $\overline{}$

Case 2: $x \in \bar{B}$. WTS $x \in \overline{A \cap B}$.

symmetric - replace A w/ B and vice versa.

Since the cases are exhaustive, the claim is proved. $\square$