Direct Proofs and Disproofs by counter-exampce

<u>Def</u> A <u>proposition</u> (statement, claim) is a statement that is either always true or always false. For a <u>proposition</u>, its <u>truth value</u> is its truth or falsity.

		prop?	T/F?
①	$2+2=4$	Y	T
②	33 is a prime number	Y	F
③	$1+2+3+4=10$	Y	Y
④	$1+2+3+4=x$	N	—
⑤	Have a great weekend!	N	—
⑥	The fastest comparison-based sorting algorithm has worst-case runtime $O(n \log n)$ for n items	Y	?? known
⑦	Every even integer greater than 2 can be written as the sum of 2 primes.	Y	?? unknown
	There is an x such that $1+2+3+4=x$	Y	

In this class, our task is to learn and practice methods of proving propositions T or F.

<u>Def</u> A <u>proof</u> is a convincing argument that a proposition is true.

A <u>disproof</u> is an argument that a proposition is false.

(Example 4.11 in book)

<u>Claim</u> If x, y are rational, then xy is rational.

<u>Step 1</u> make sure we understand claim.

rational : $x = \frac{n}{d}$ where n, d integers $d \neq 0$

integer : $\ldots, -3, -2, -1, 0, 1, 2, \ldots$

examples of rational #s :

$1/2$ is rational because let $n = 1$, $d = 2$

0.5 is rational because $0.5 = 1/2$

is $\sqrt{2}$ rational? no

-10 is rational because $\frac{-10}{1}$ $-10 = n$, $1 = d$

<u>Step 2</u> test it! do some examples

x	y	xy	x, y rational	xy rat.
$1/2$	$1/2$	$1/4$	T	T
π	2	2π	F	F

<u>Claim</u> If x, y are rational, then xy is rational.

<u>Proof</u> Start by assuming x, y rational.

<u>statement</u>	<u>reasoning</u>
$x = \frac{n_x}{d_x}$, $y = \frac{n_y}{d_y}$ where n_x, d_x, n_y, d_y are integers and $d_x, d_y \neq 0$	by def. of rational x, y rational
$xy = \frac{n_x n_y}{d_x d_y}$	by substitution
$xy = \frac{n}{d_x d_y}$ where n is int	because product of ints is int
$xy = \frac{n}{d}$ where d is a nonzero int	because product of nonzero ints is a nonzero int
xy is rational	by def. of rational

$\square$

Def A direct proof starts from known facts or defs and repeatedly applies logical deduction to derive new facts and end up with the claim.

Q: Is the converse true?

if p then q (if $\underbrace{x, y \text{ rat.}}_{p}$, then $\underbrace{xy \text{ rat.}}_{q}$)

if q then p

Converse: if xy rational, then x, y rat.

False.

We disprove the claim with a counter-example. Give an x, y so that xy is rational, but x, y not both rational.

$x = \sqrt{2}$ $\quad y = \sqrt{2}$ $\quad xy = \sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2$

not rat. $\qquad$ rational

Def A disproof by counter-example constructs an example for which the claim is false and explains why it is false.