Welcome back!

Today:

- short lecture; don't worry about taking notes if it feels fast
- group activity

Tomorrow:

- continuation of group activity

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*algorithms are represented using **pseudocode**, a mix of precise/unambiguous notation and words*

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Idea #1: implement the algorithm, run it, time it...

- depends on software, hardware, operating system, etc.
- implementation takes time and is error-prone
- how do we choose which inputs to run it on?

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Idea #2: find a function that expresses runtime in terms of input size

- runtime: number of primitive operations (arithmetic operations, logical operations, variable retrieval, variable assignment, etc.)

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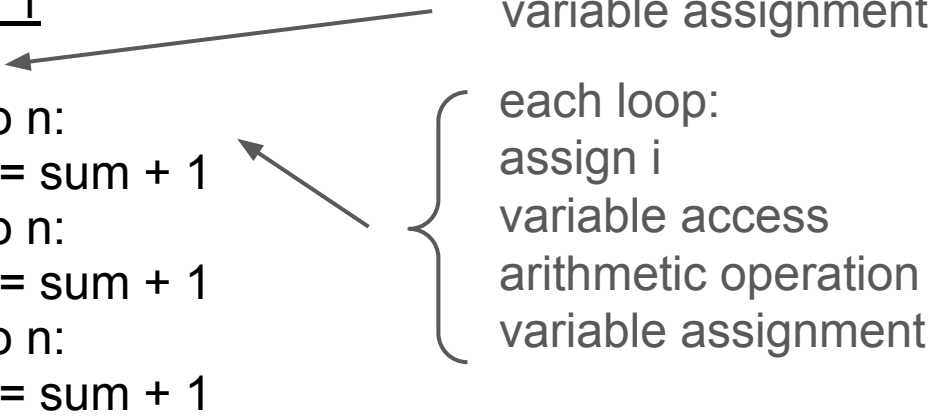

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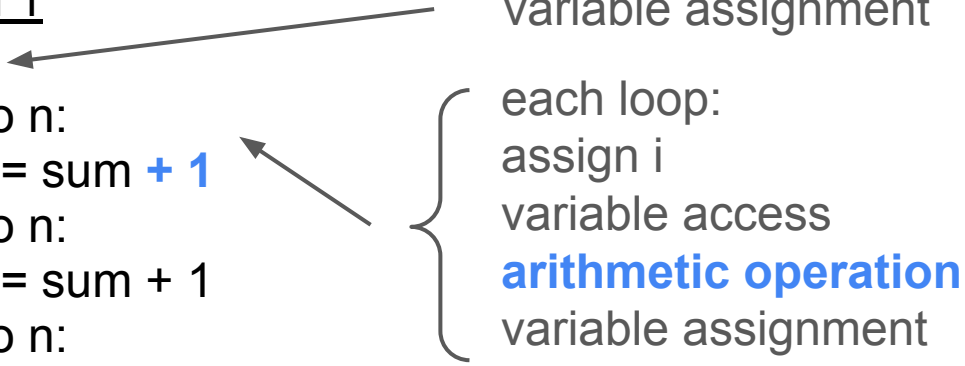

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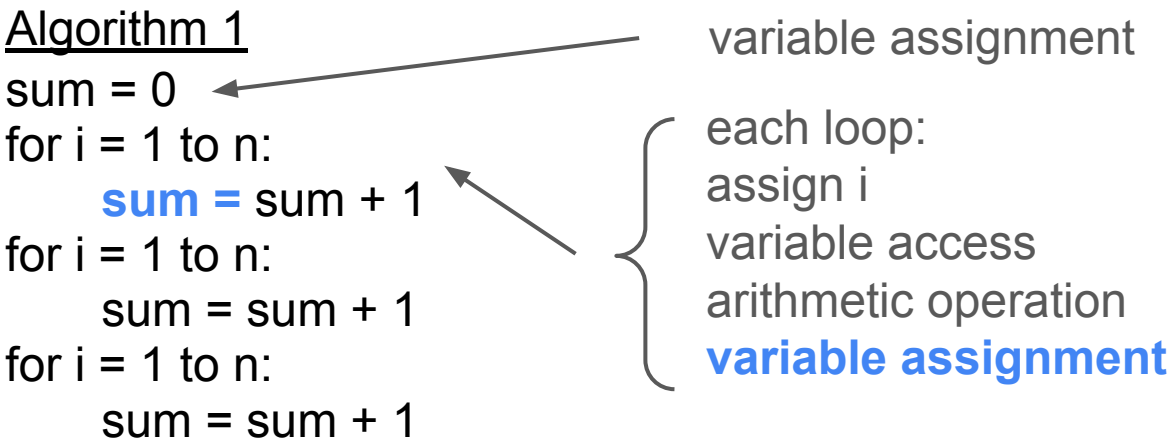
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$1+3*n*4 = 12n+1$

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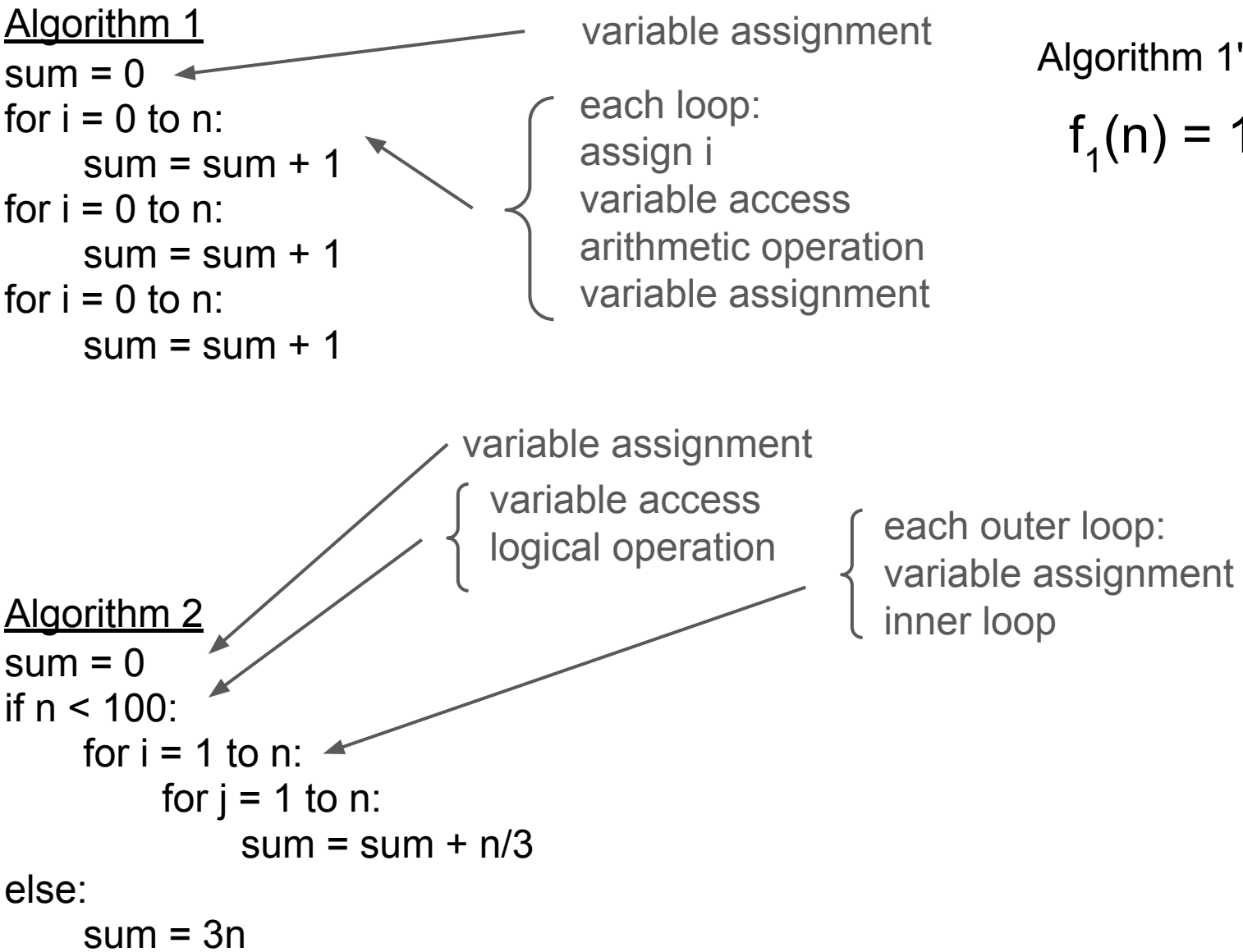
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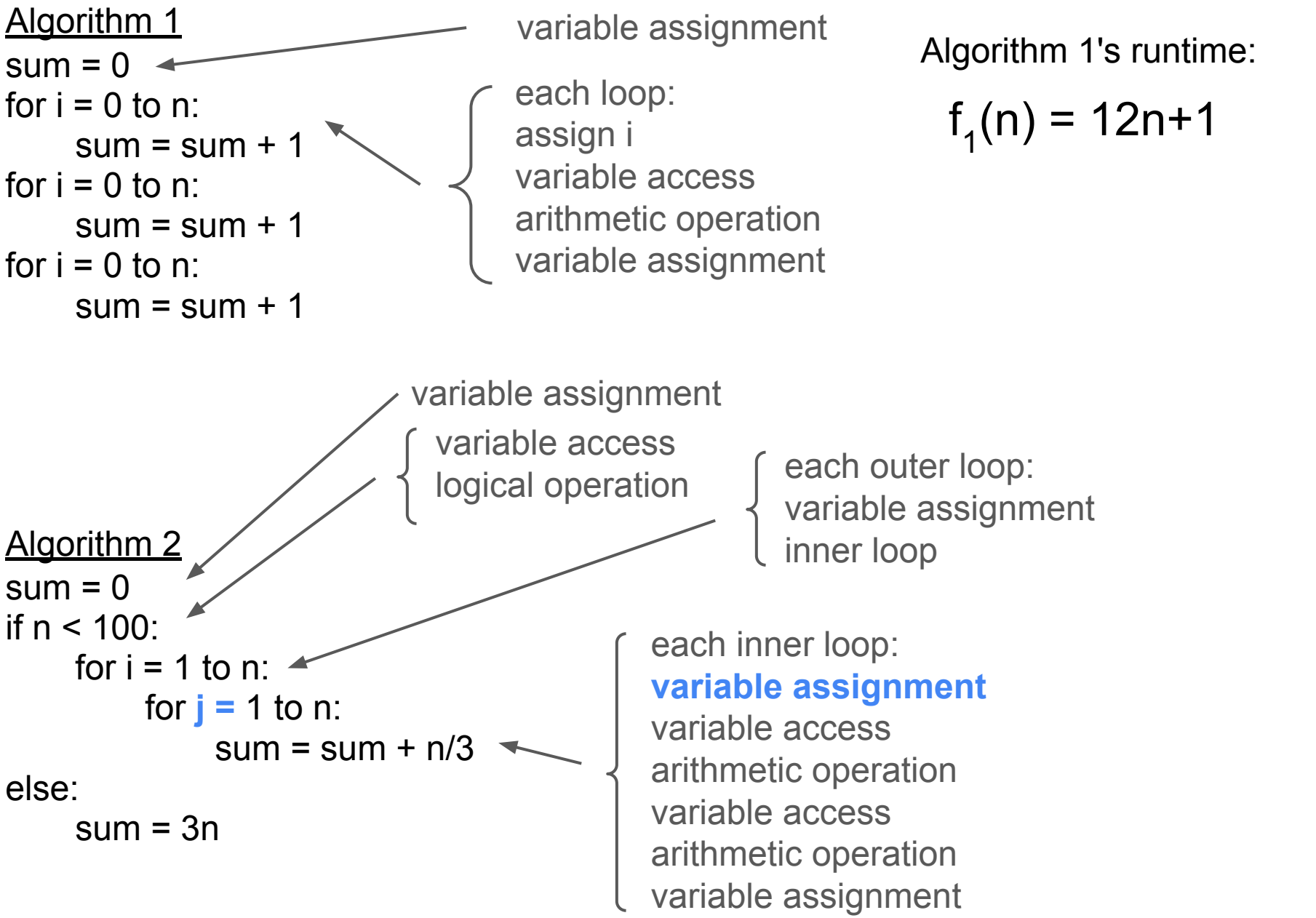
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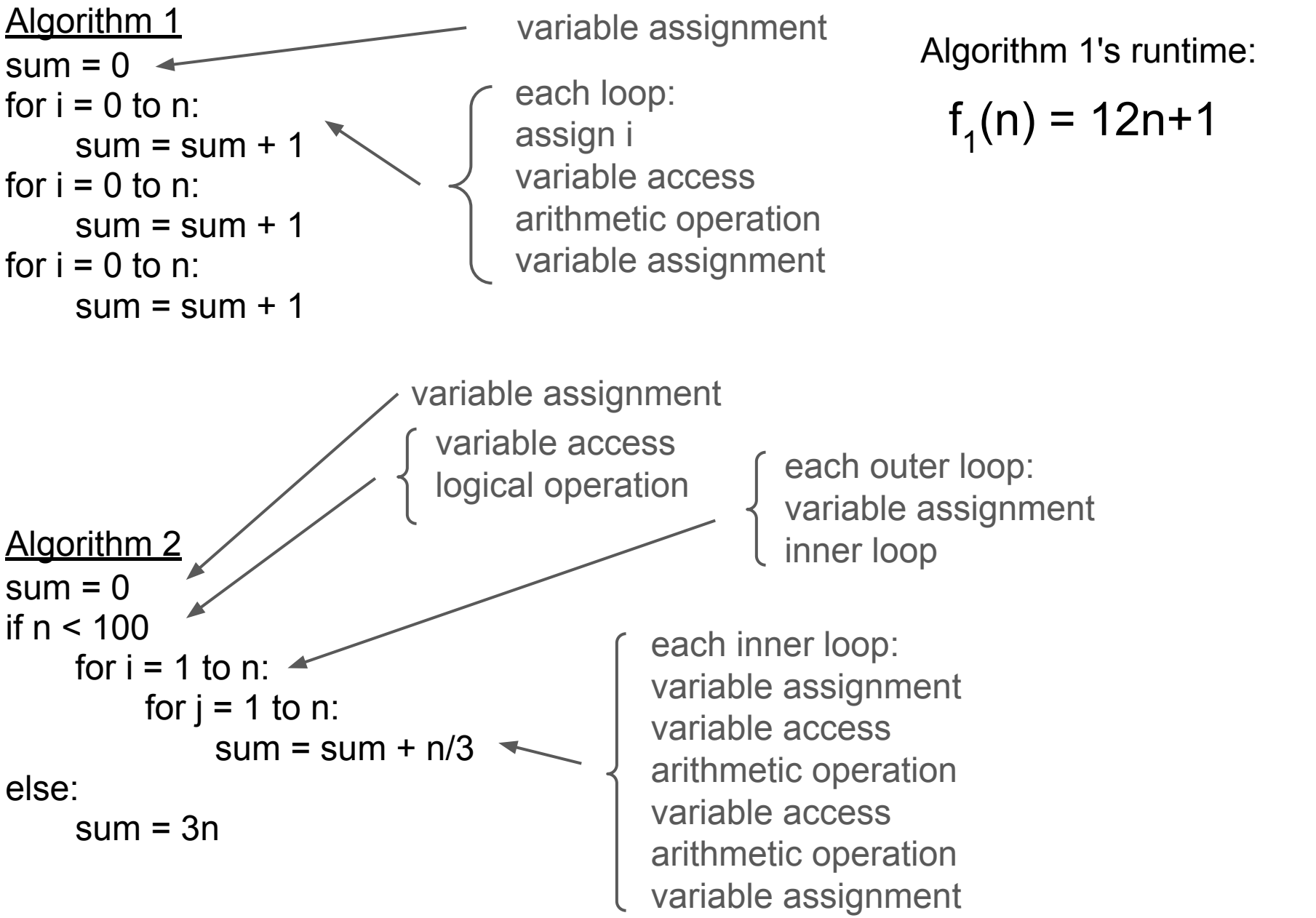
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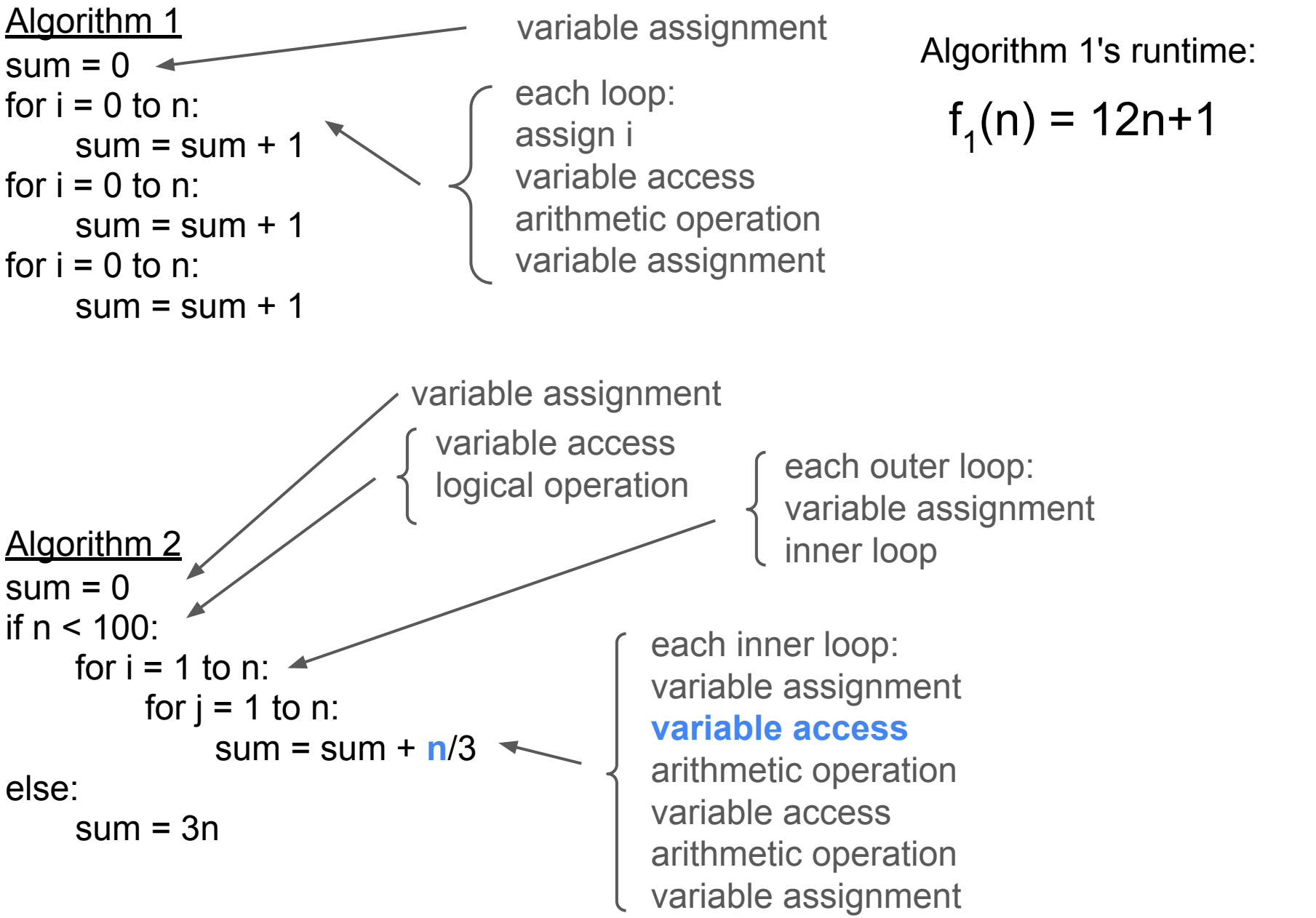
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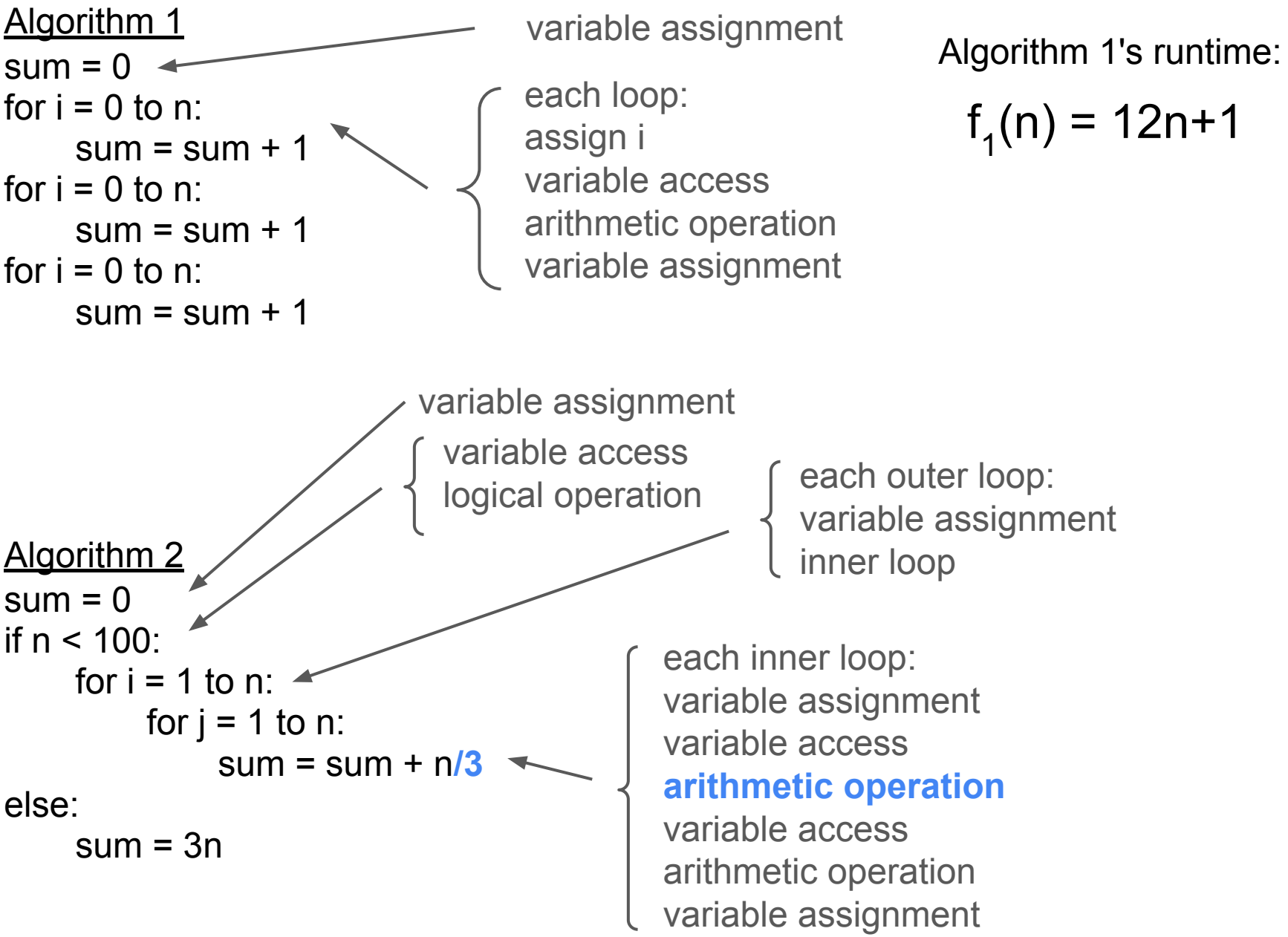
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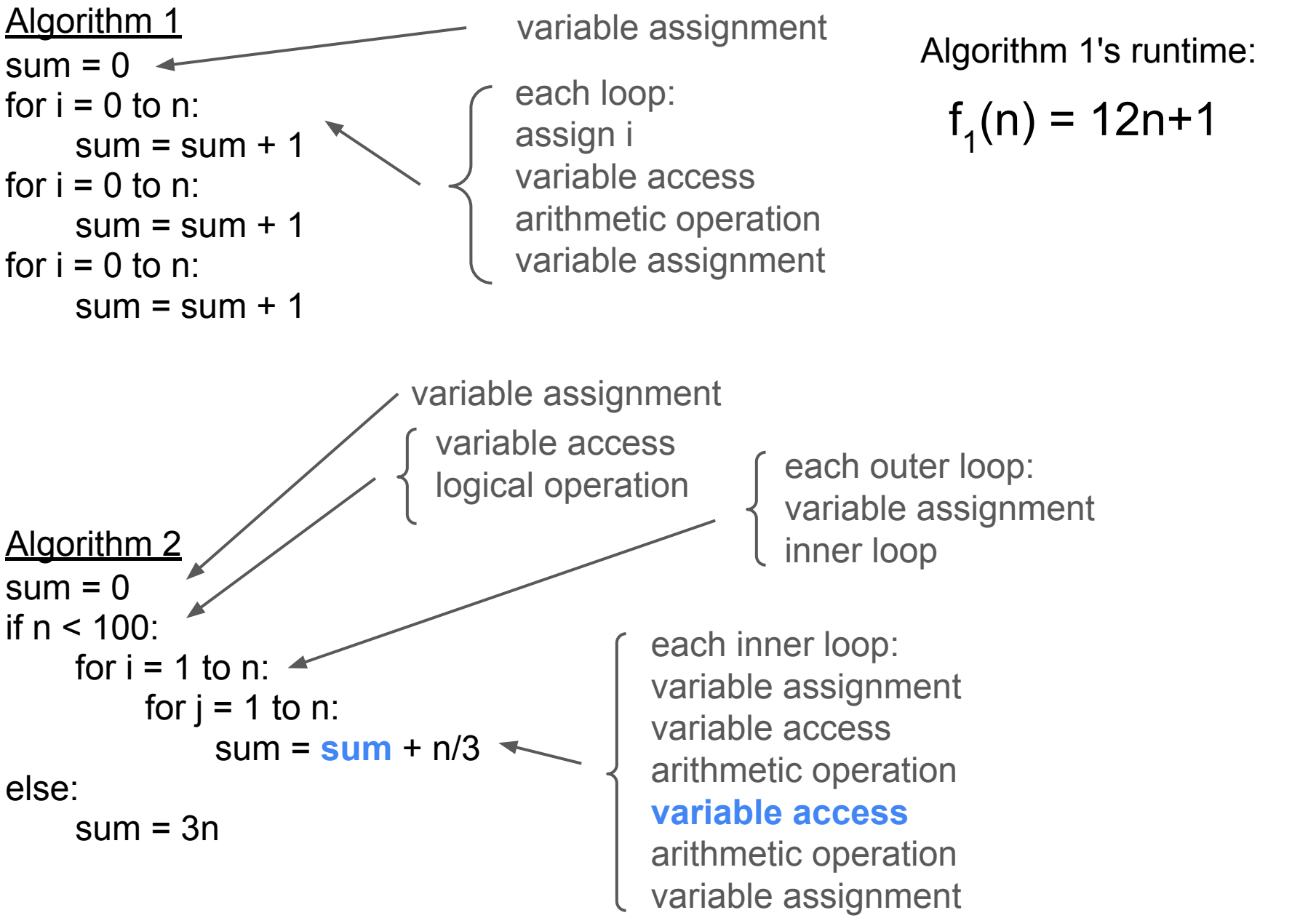
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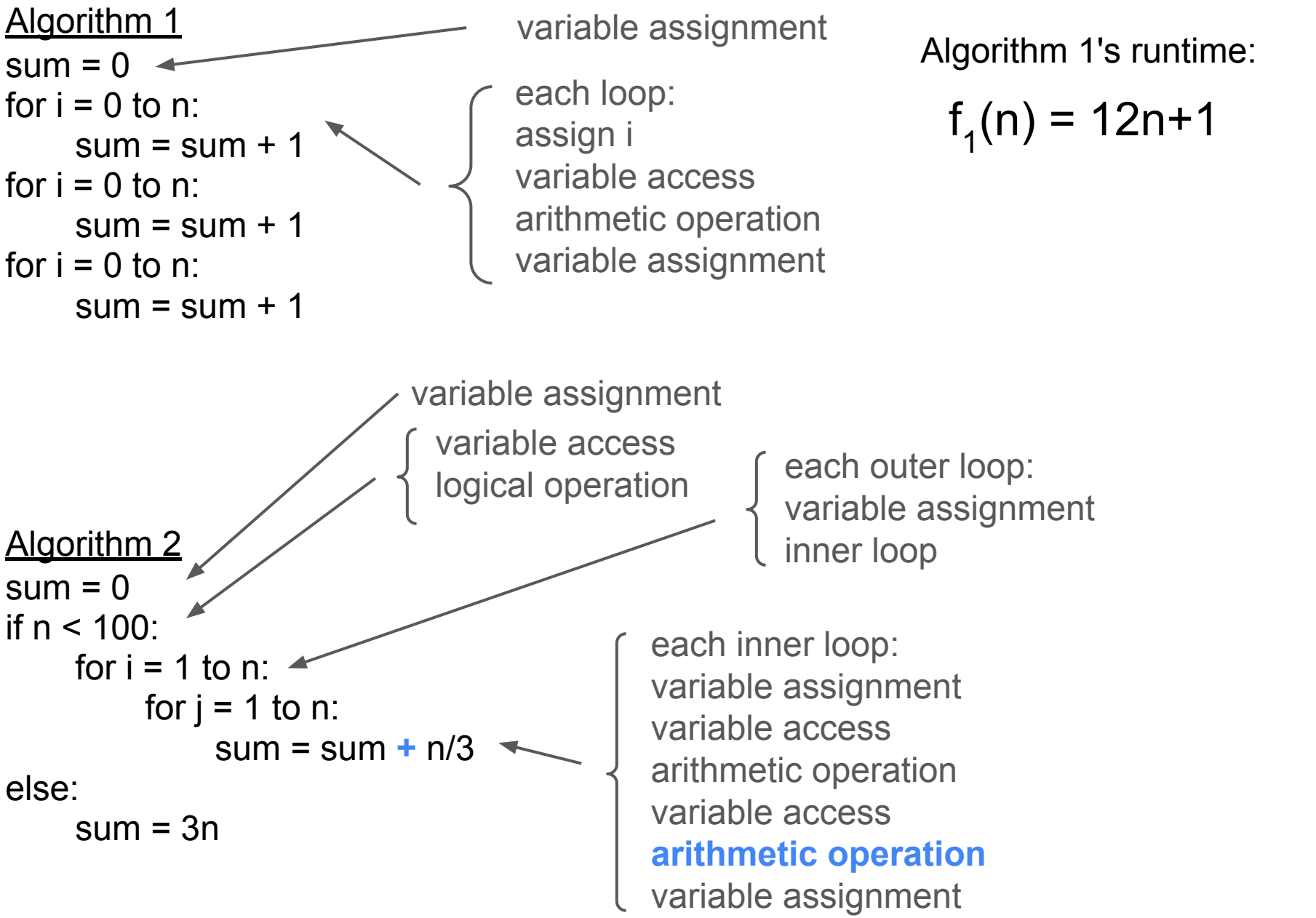
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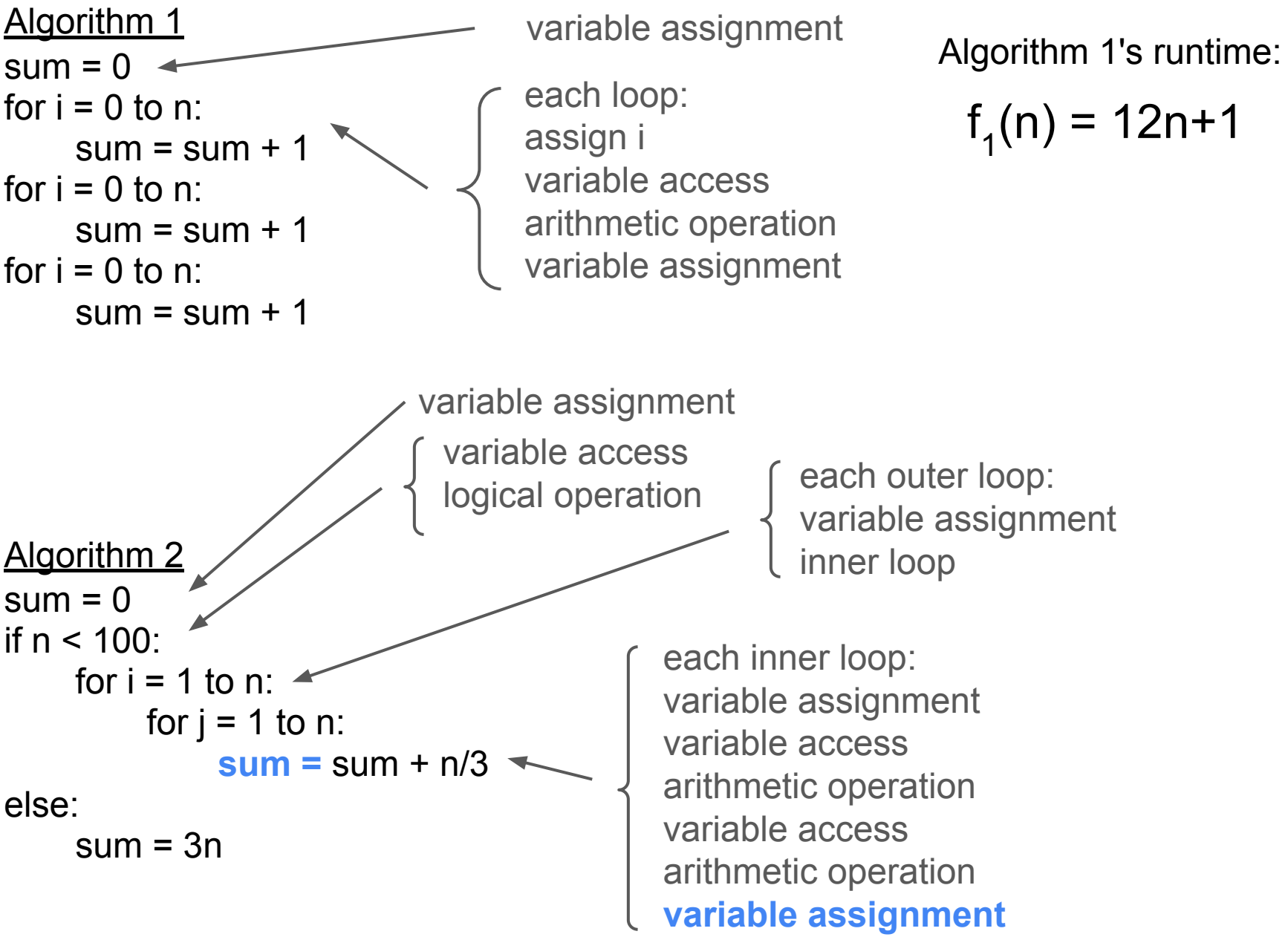
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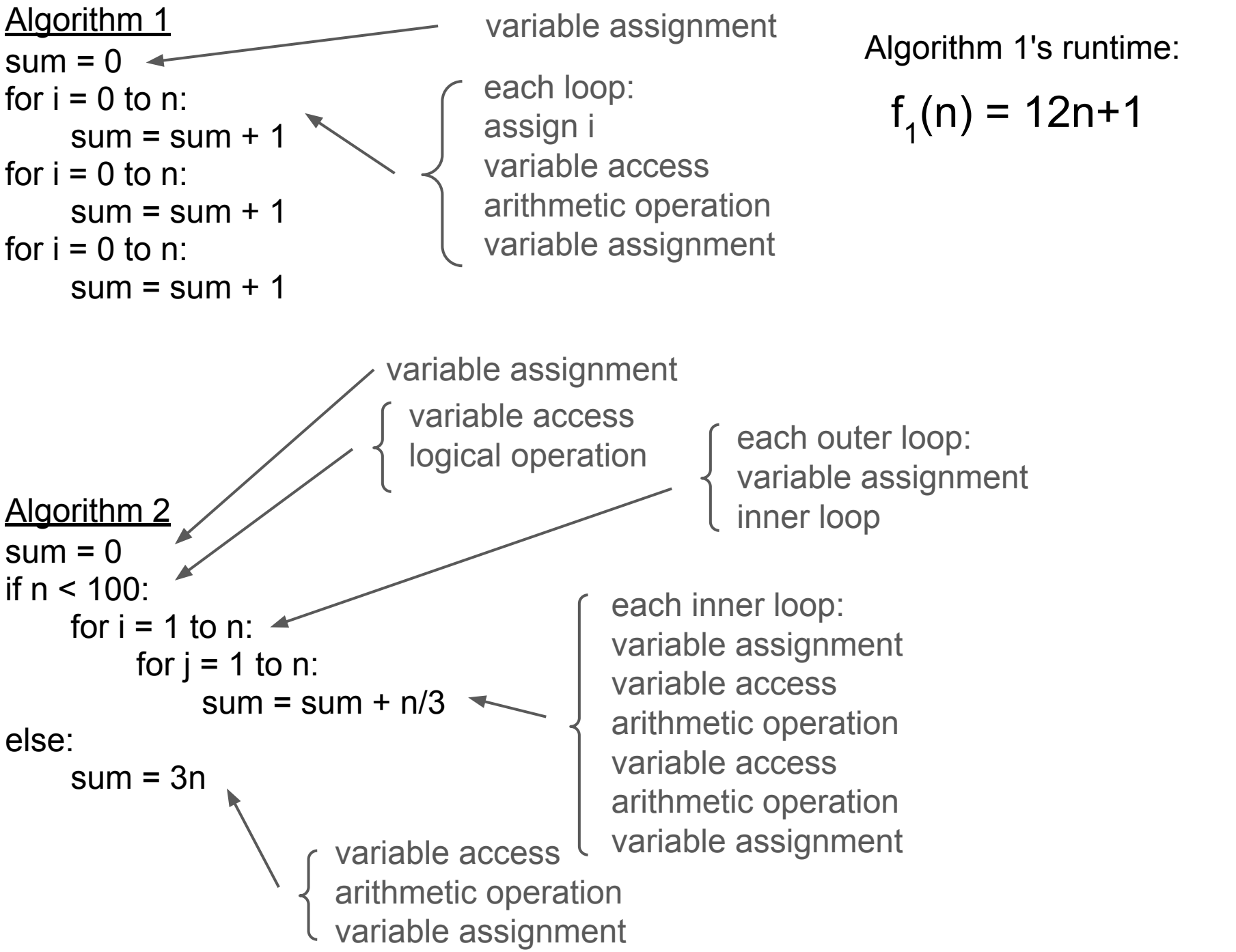
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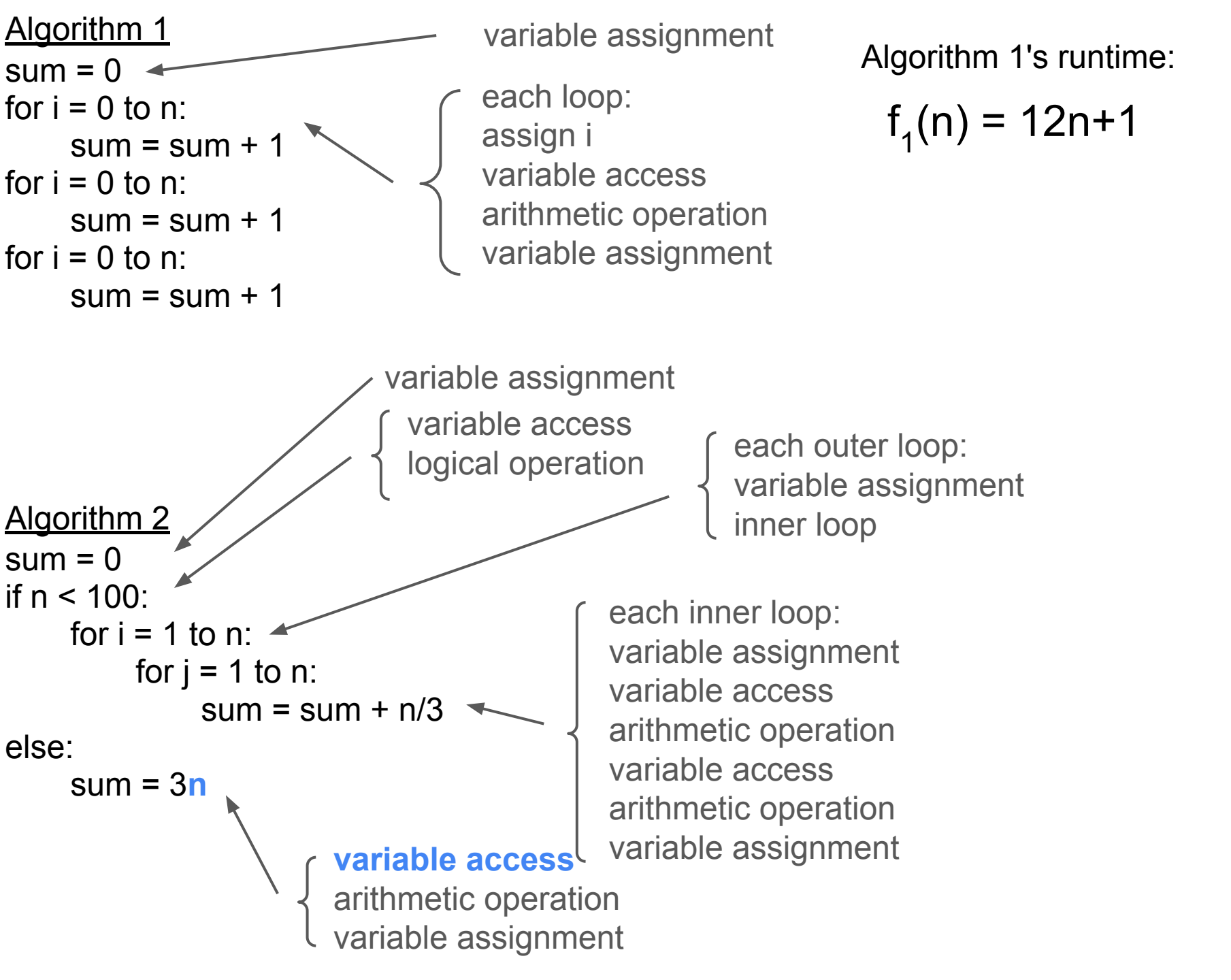
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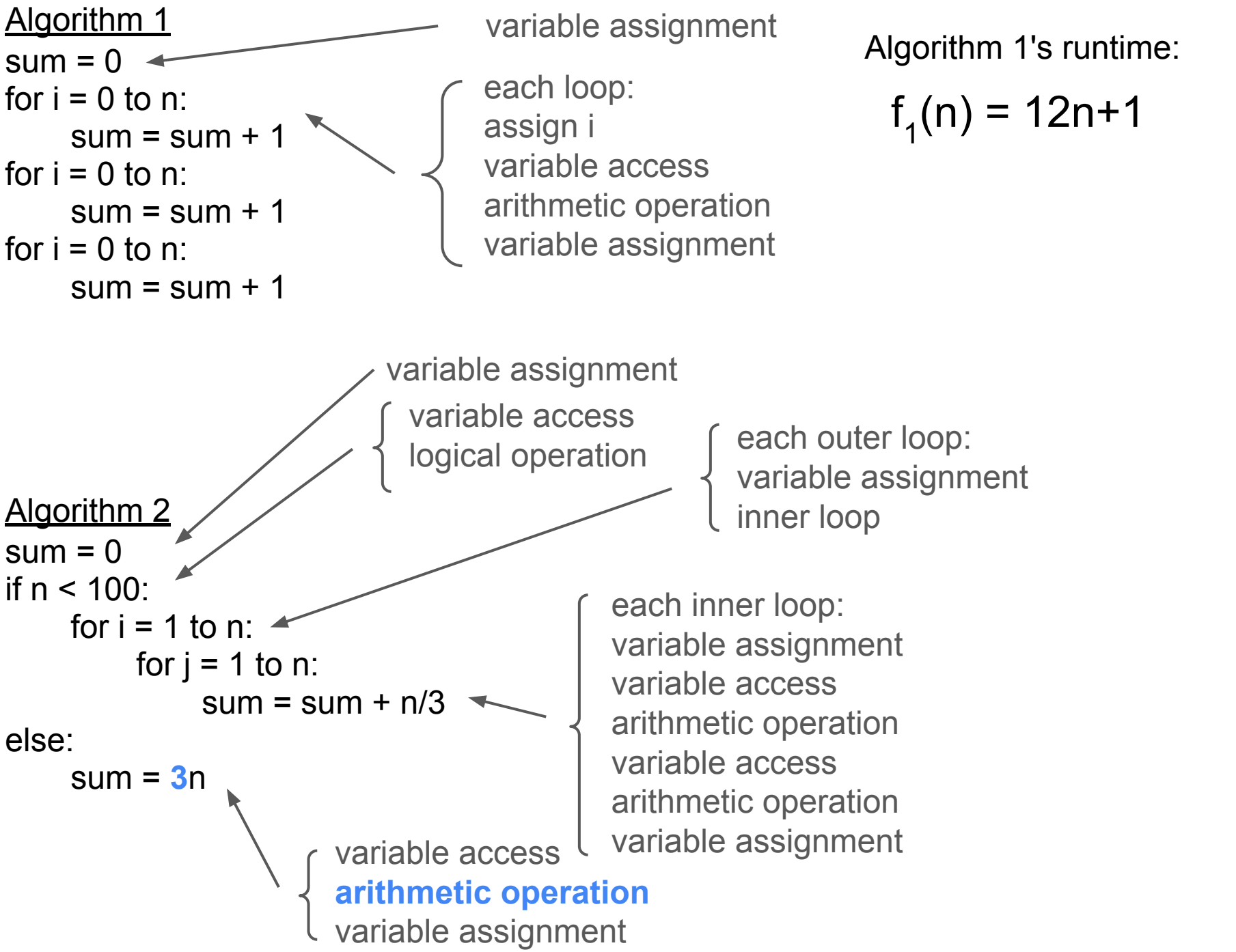
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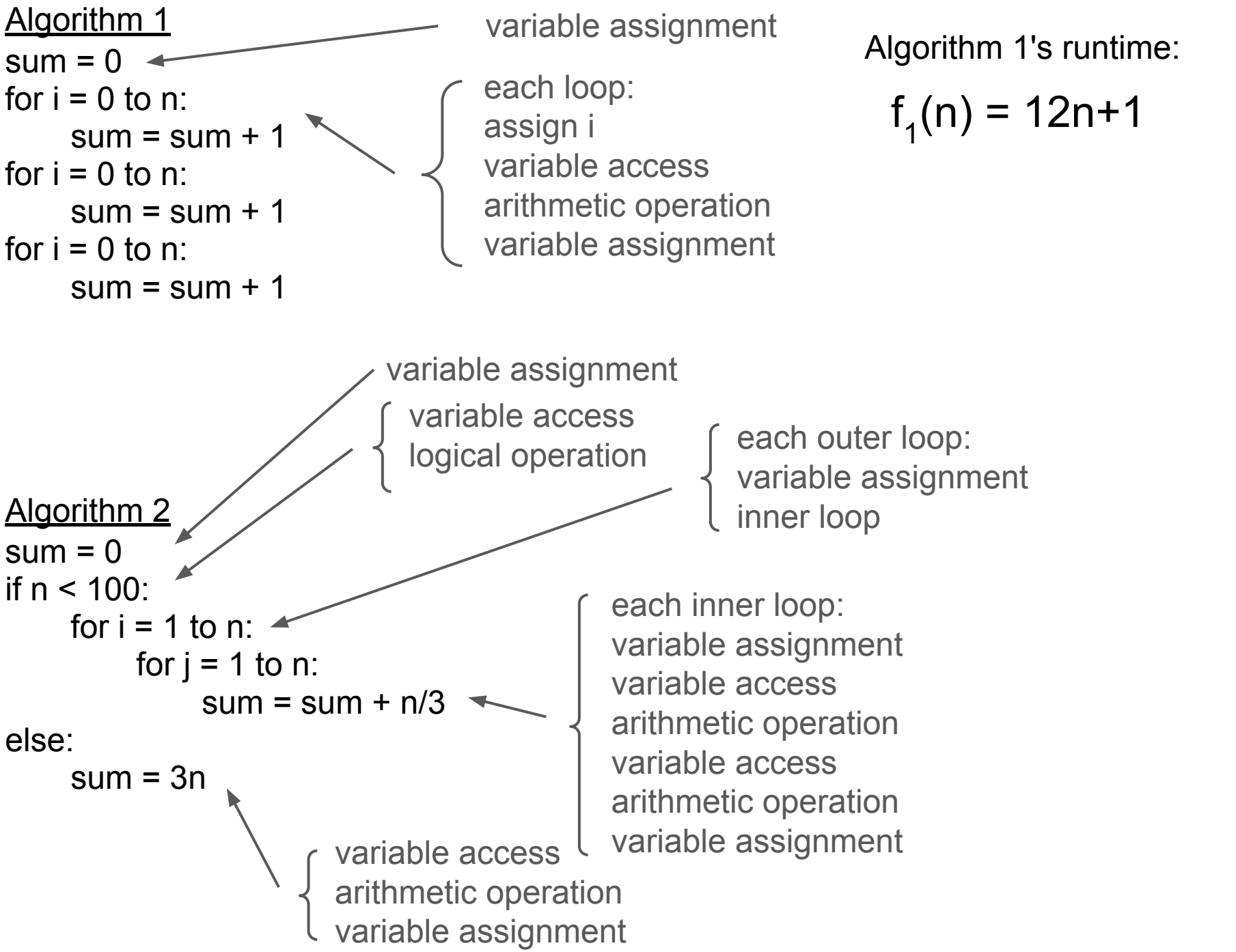
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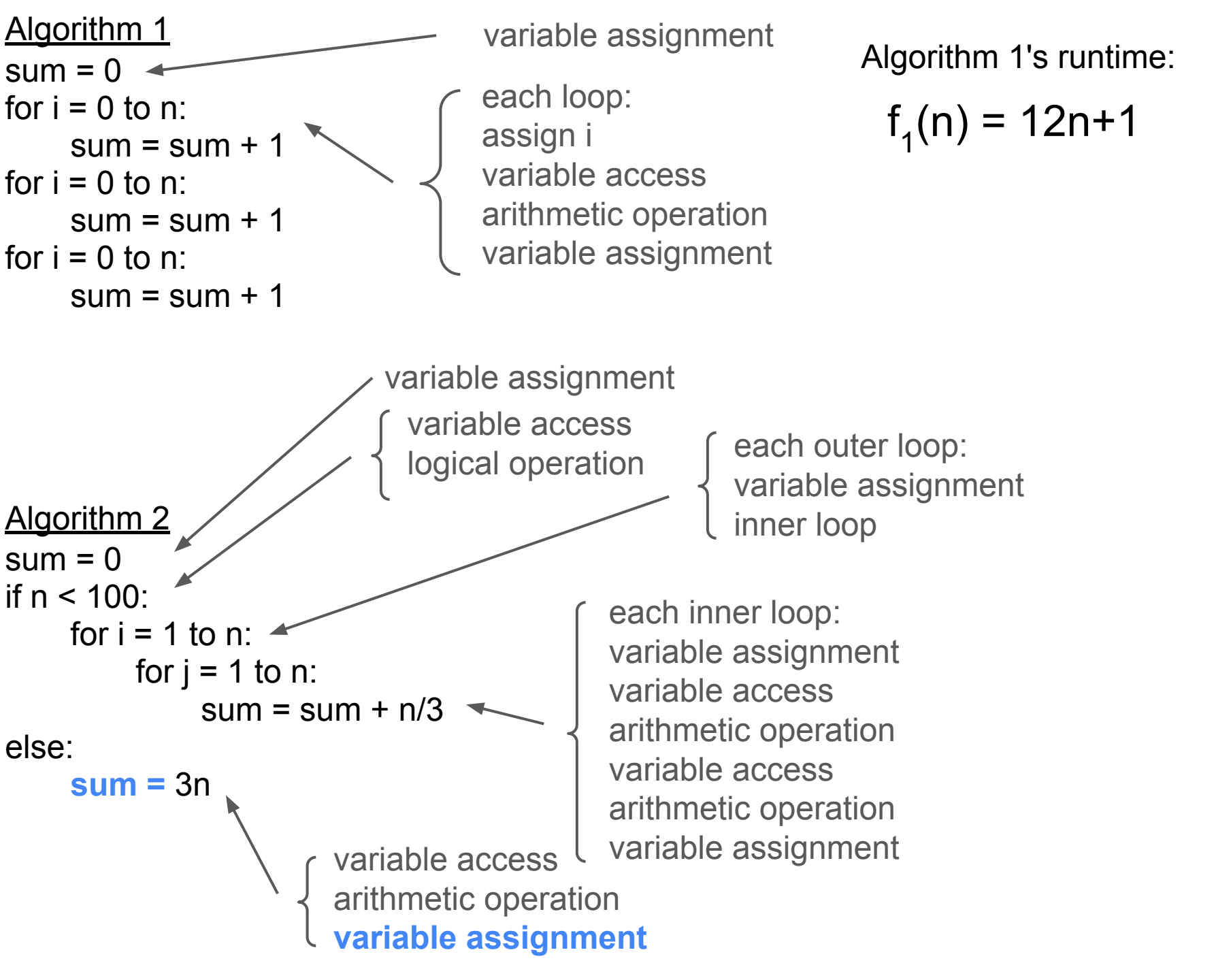
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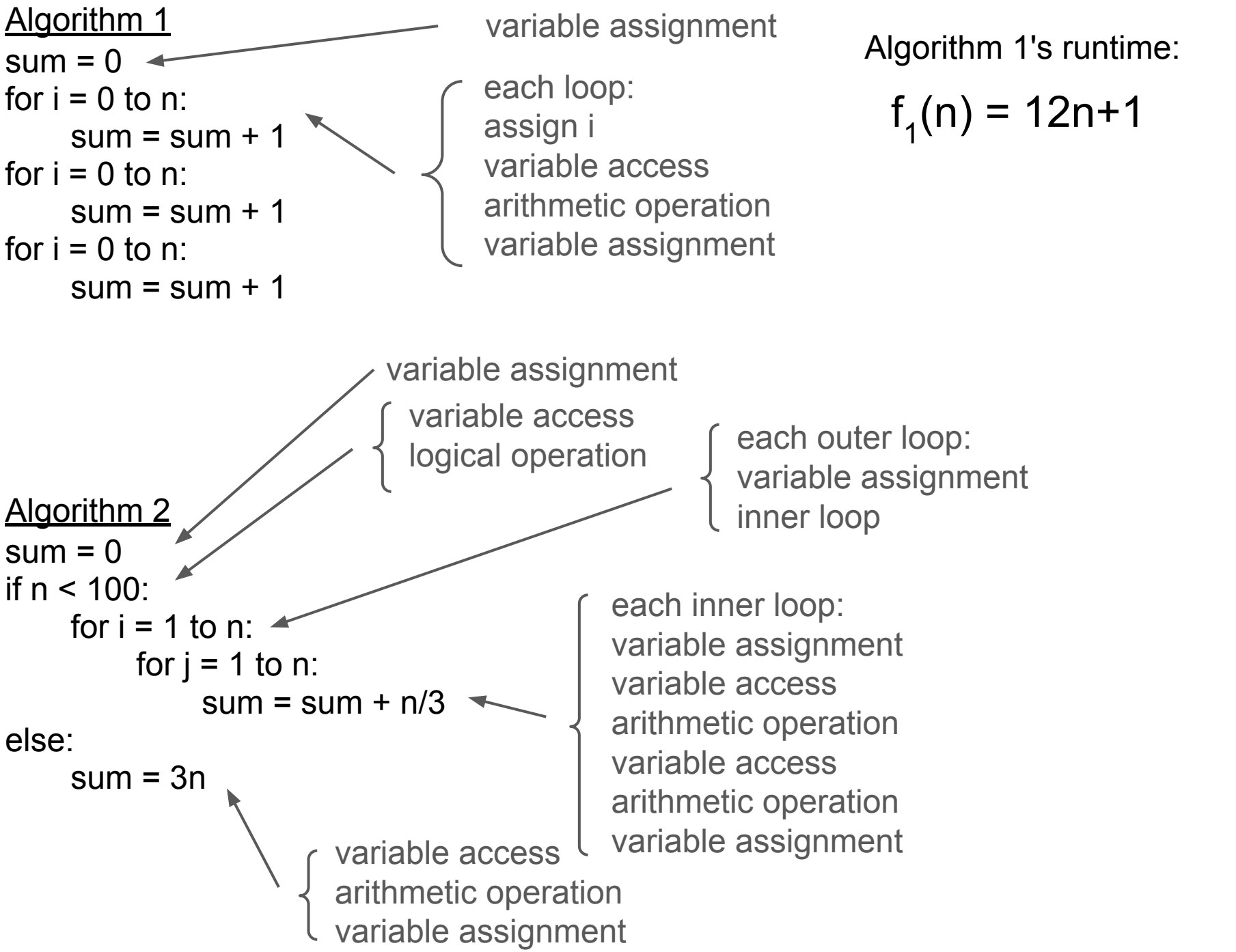
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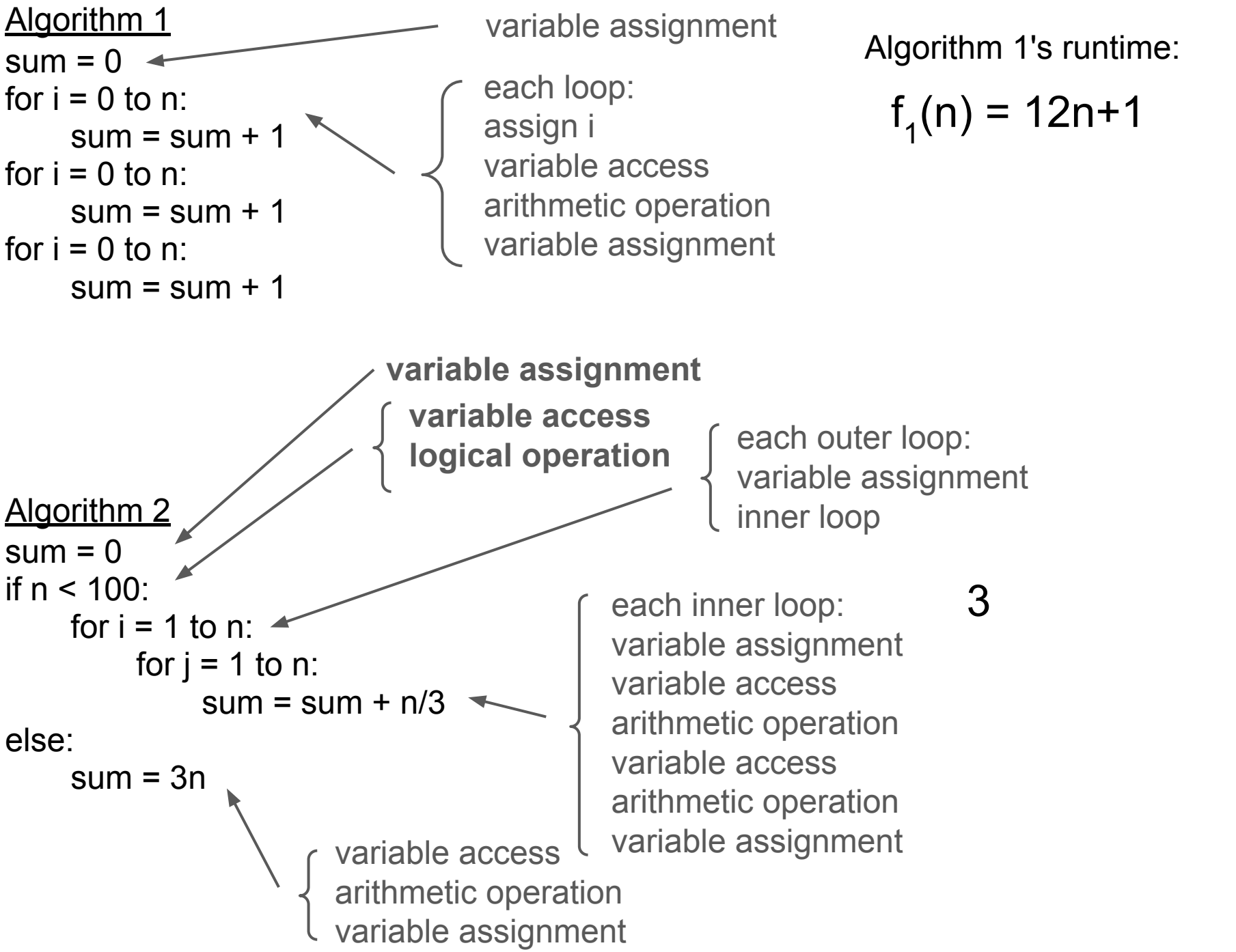
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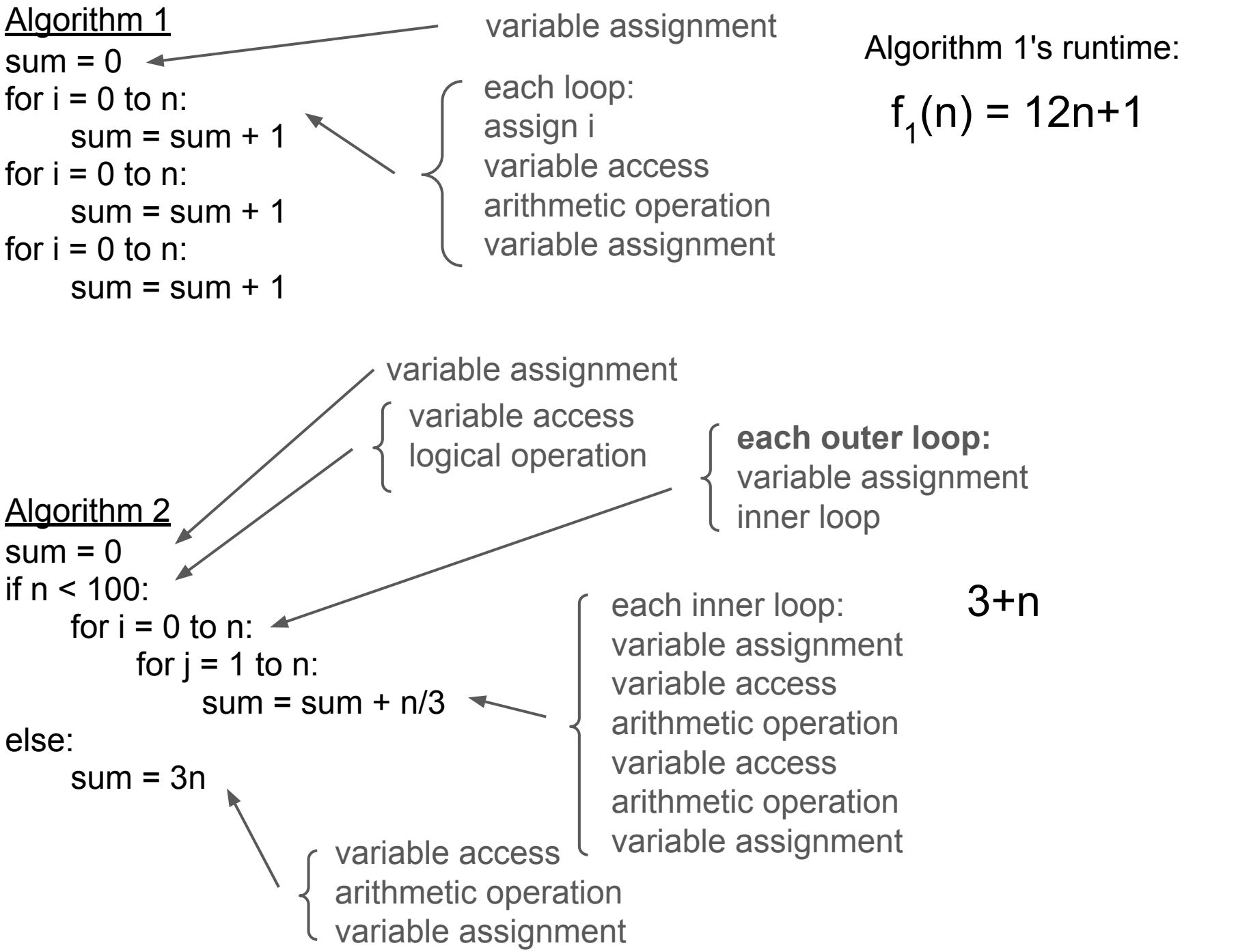
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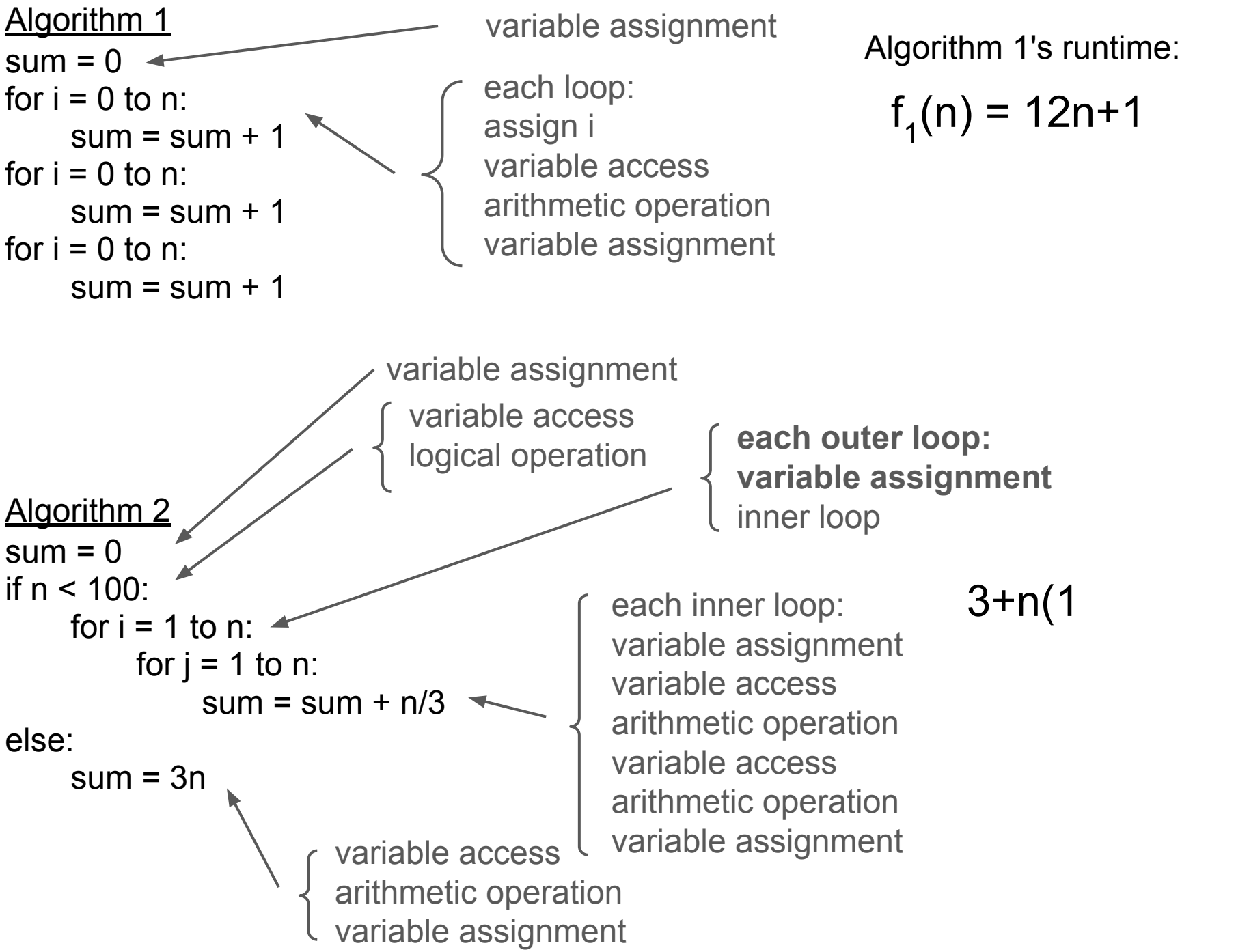
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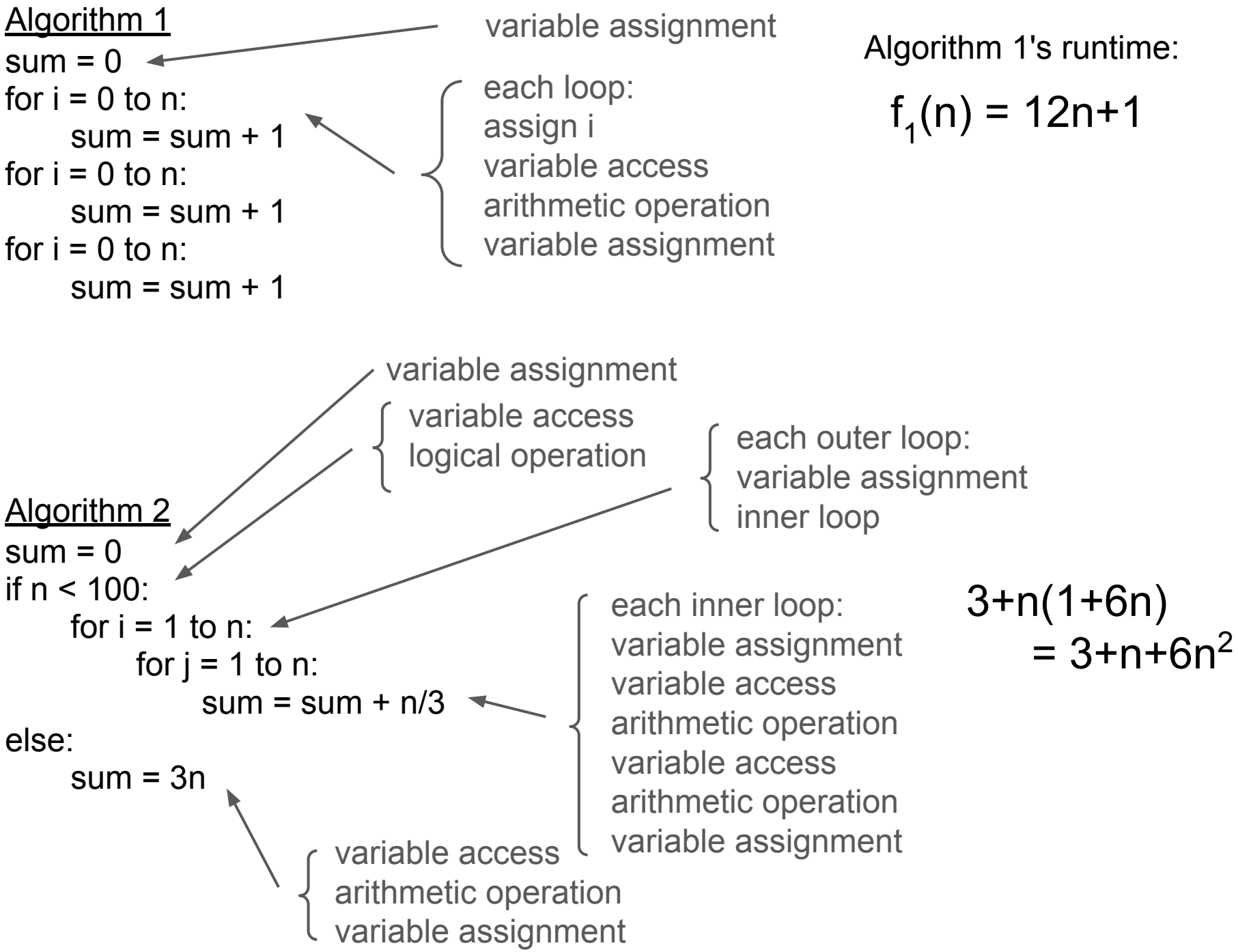
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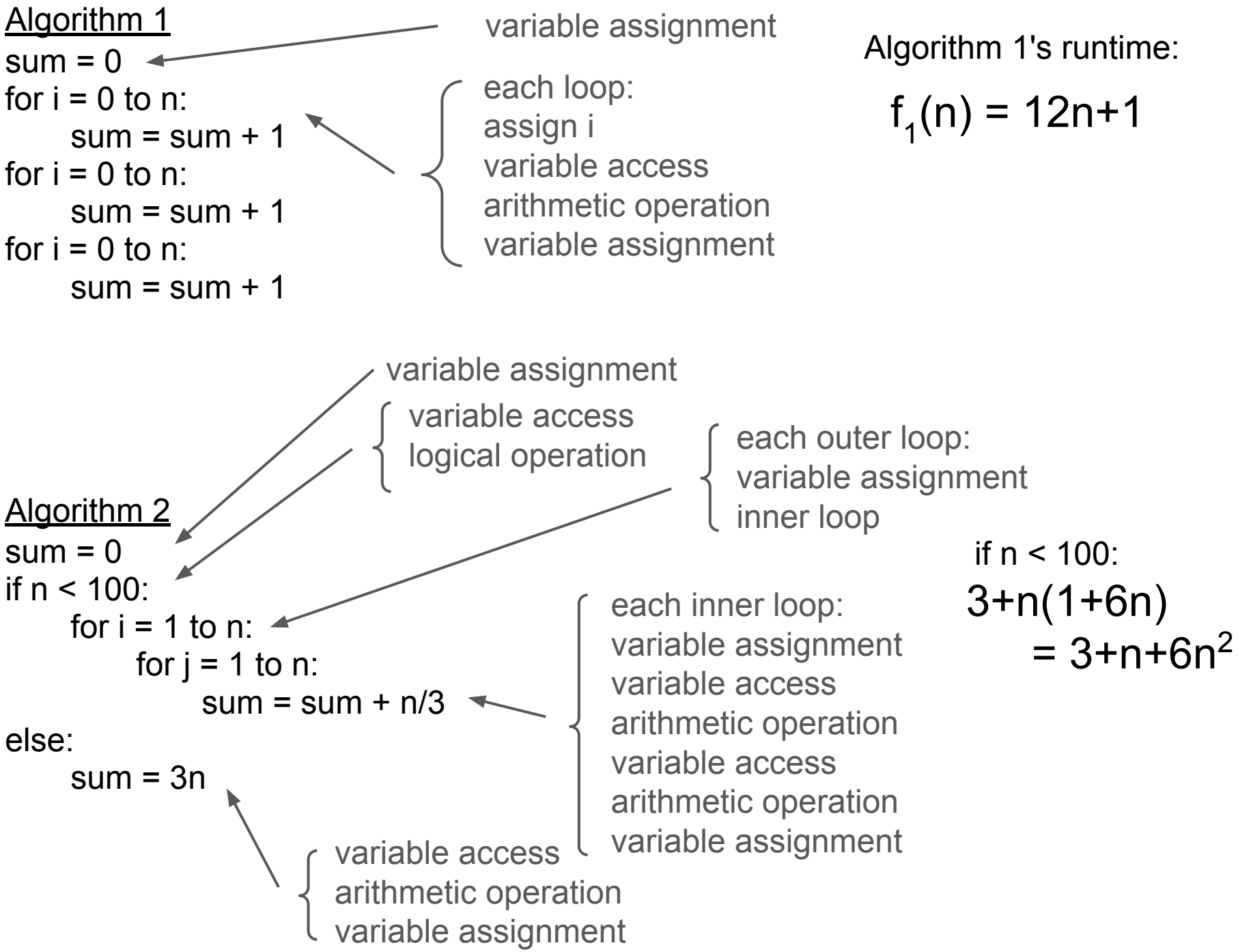
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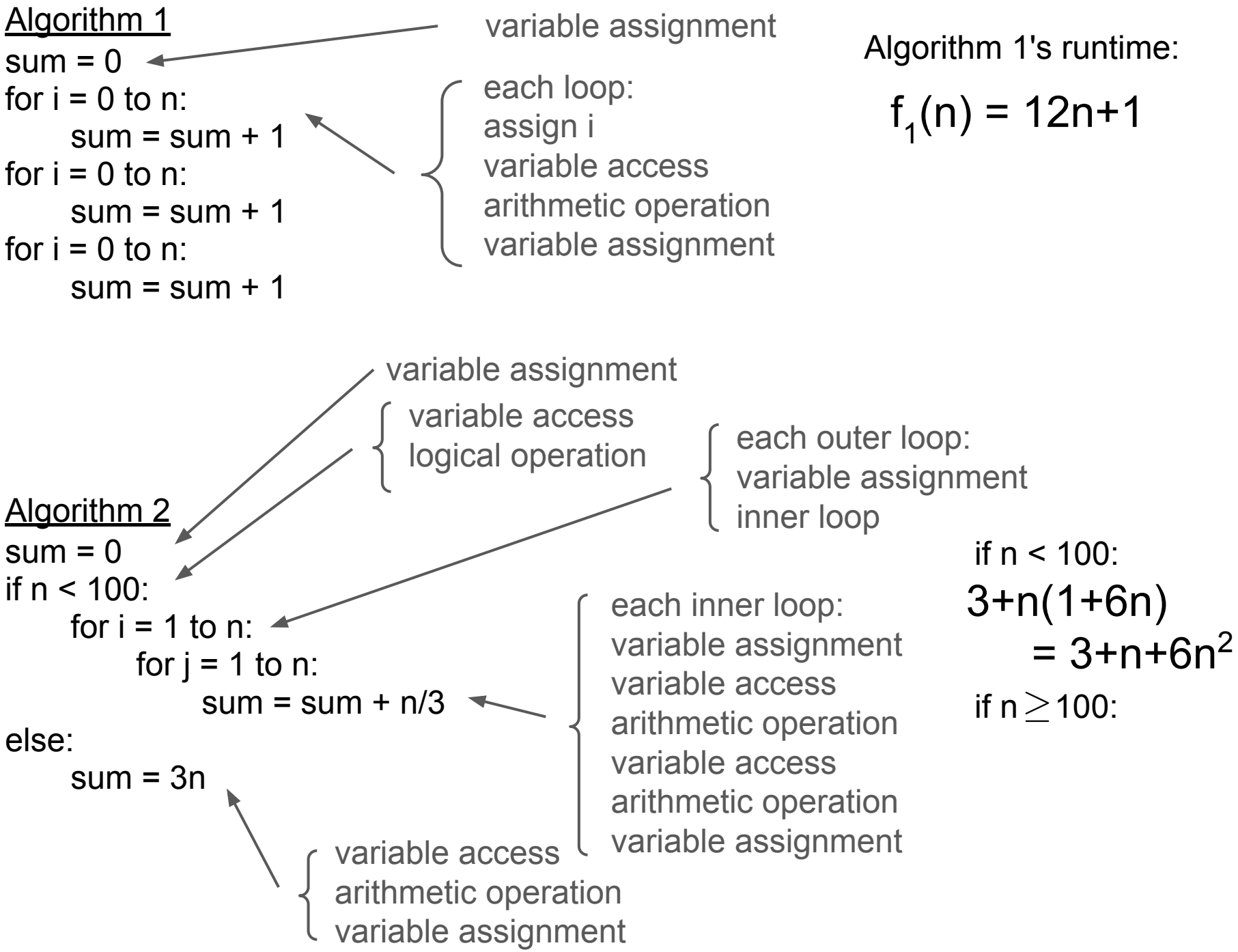
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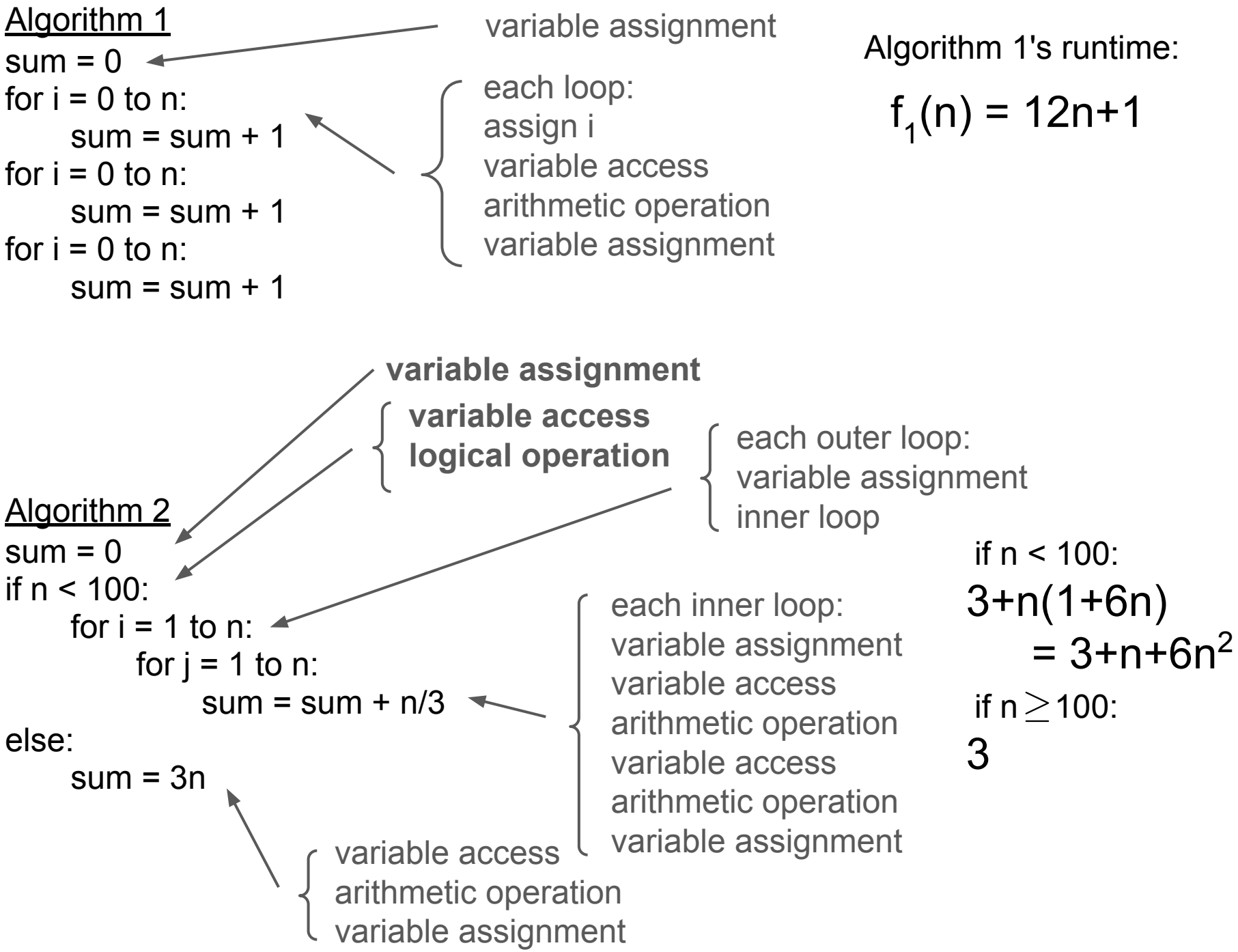
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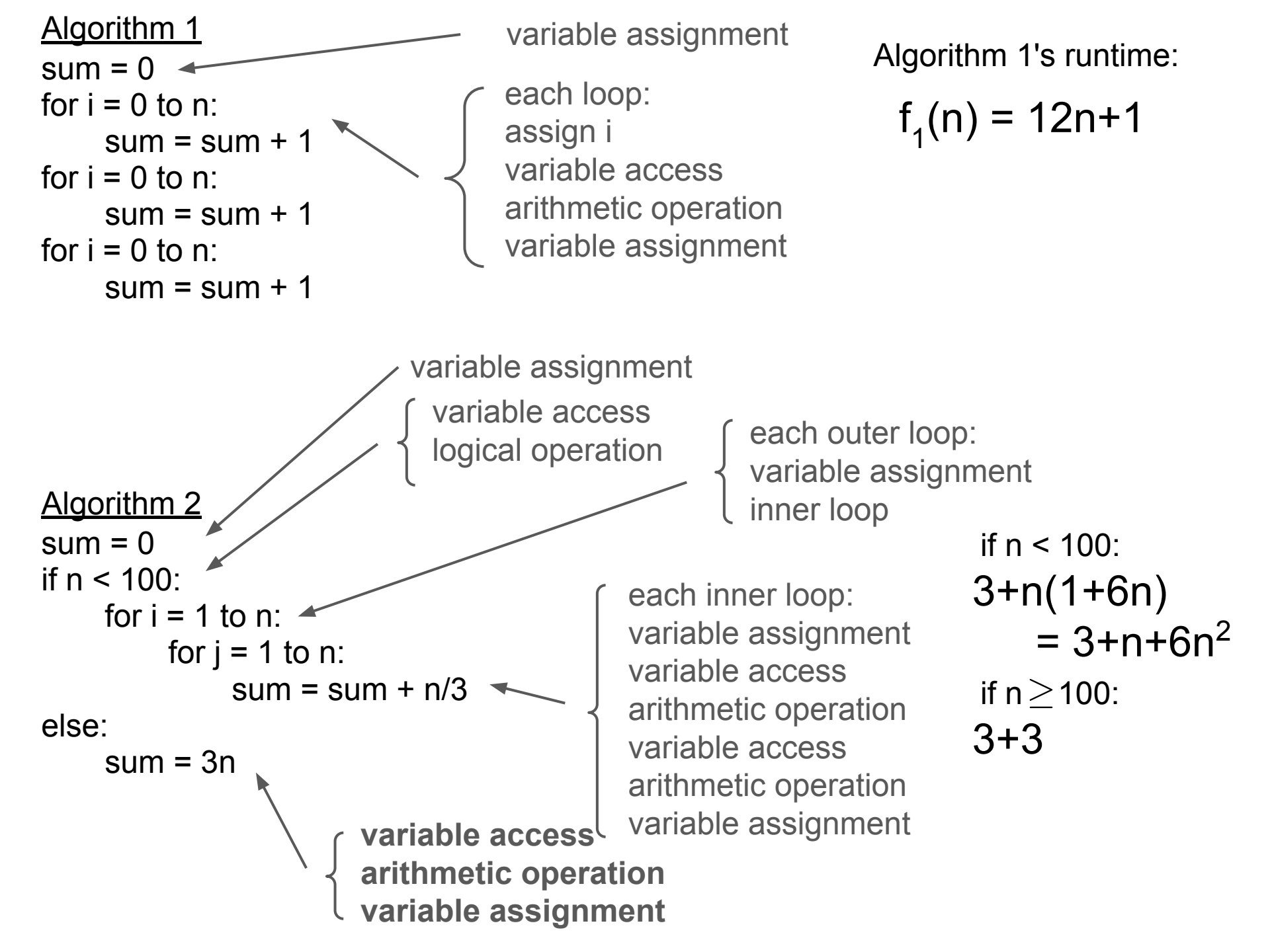
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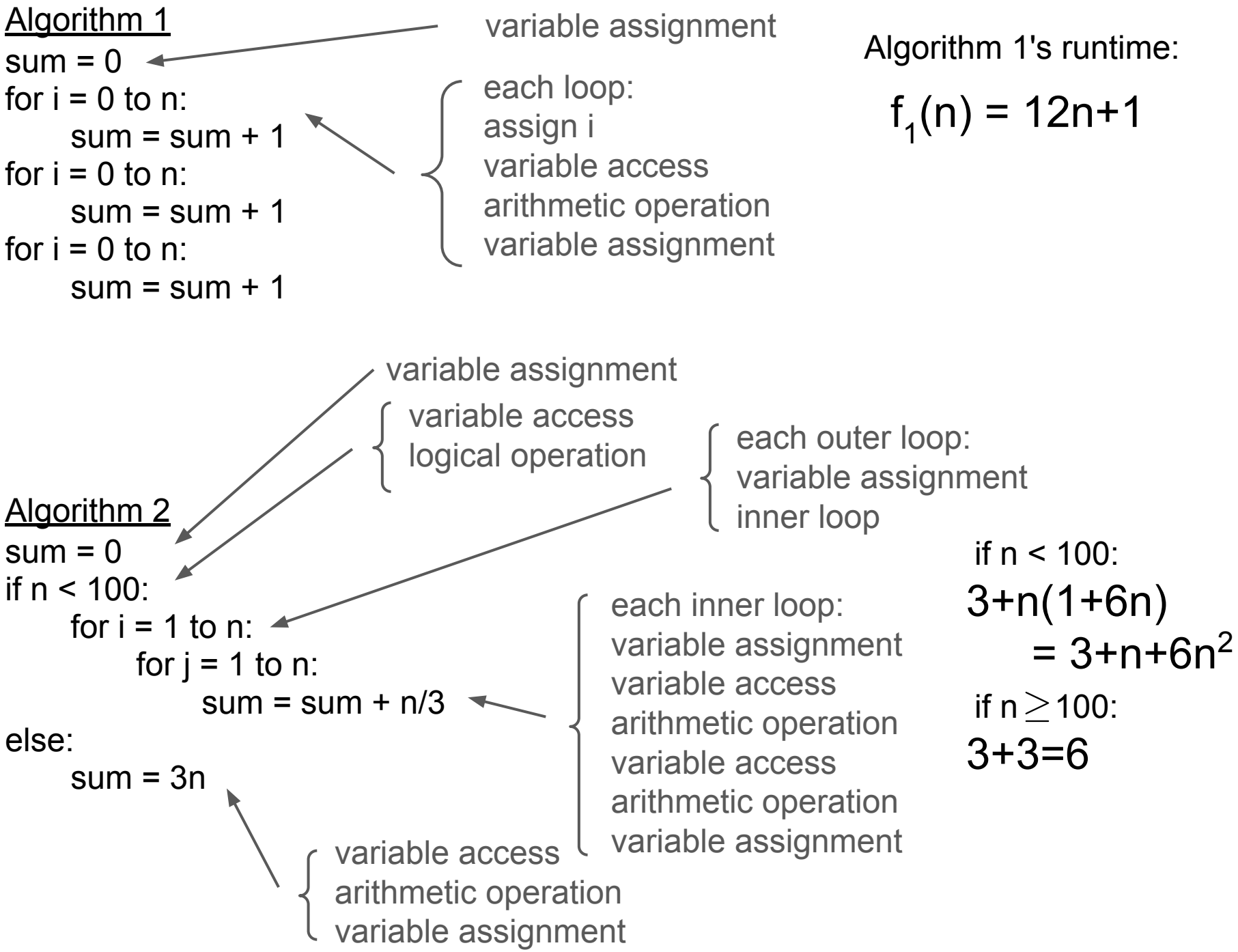
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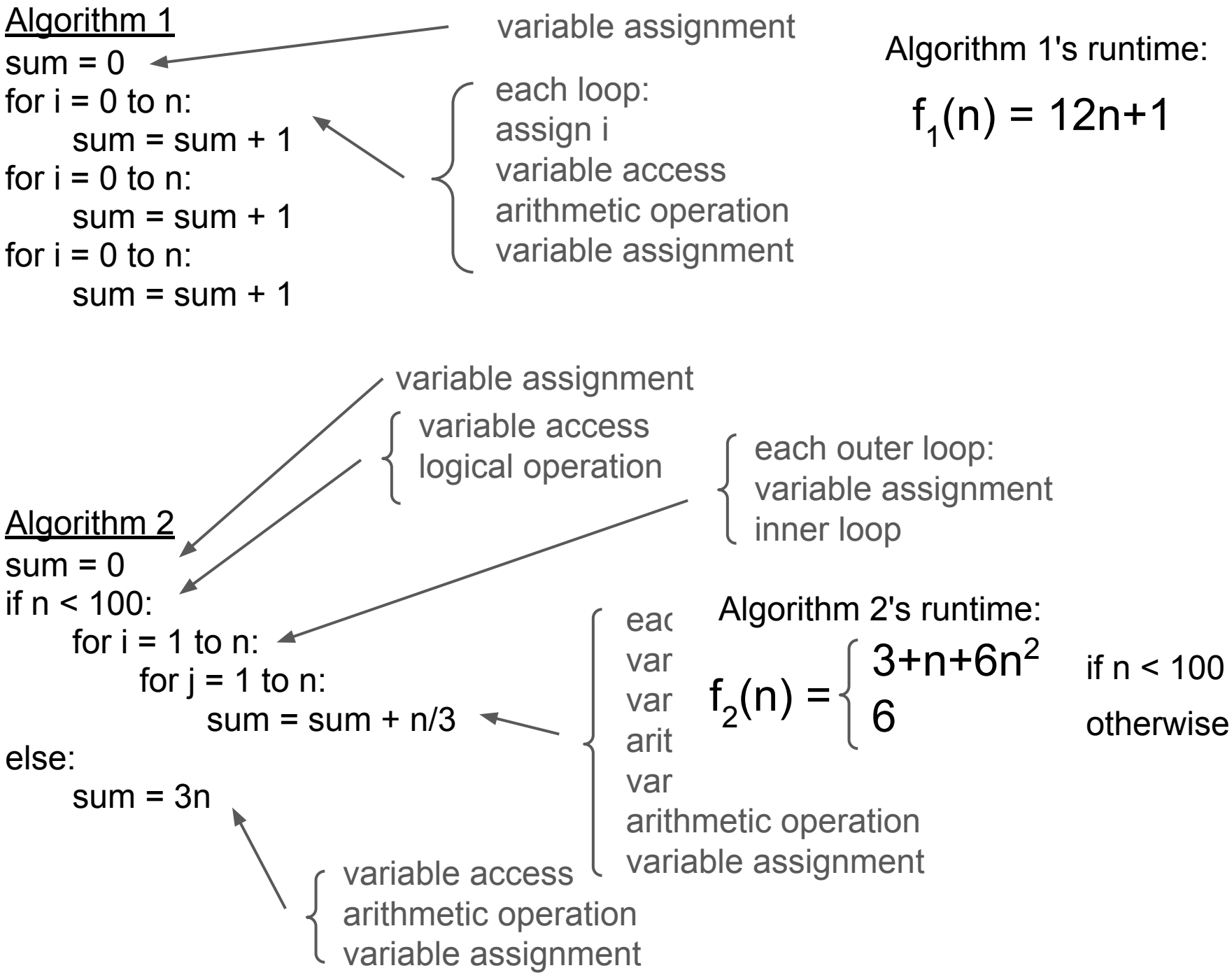
if $n \geq 100$:
3+3=6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Algorithm 2's runtime:
$f_2(n) = \begin{cases} 3+n+6n^2 & \text{if } n < 100 \\ 6 & \text{otherwise} \end{cases}$
arithmetic operation
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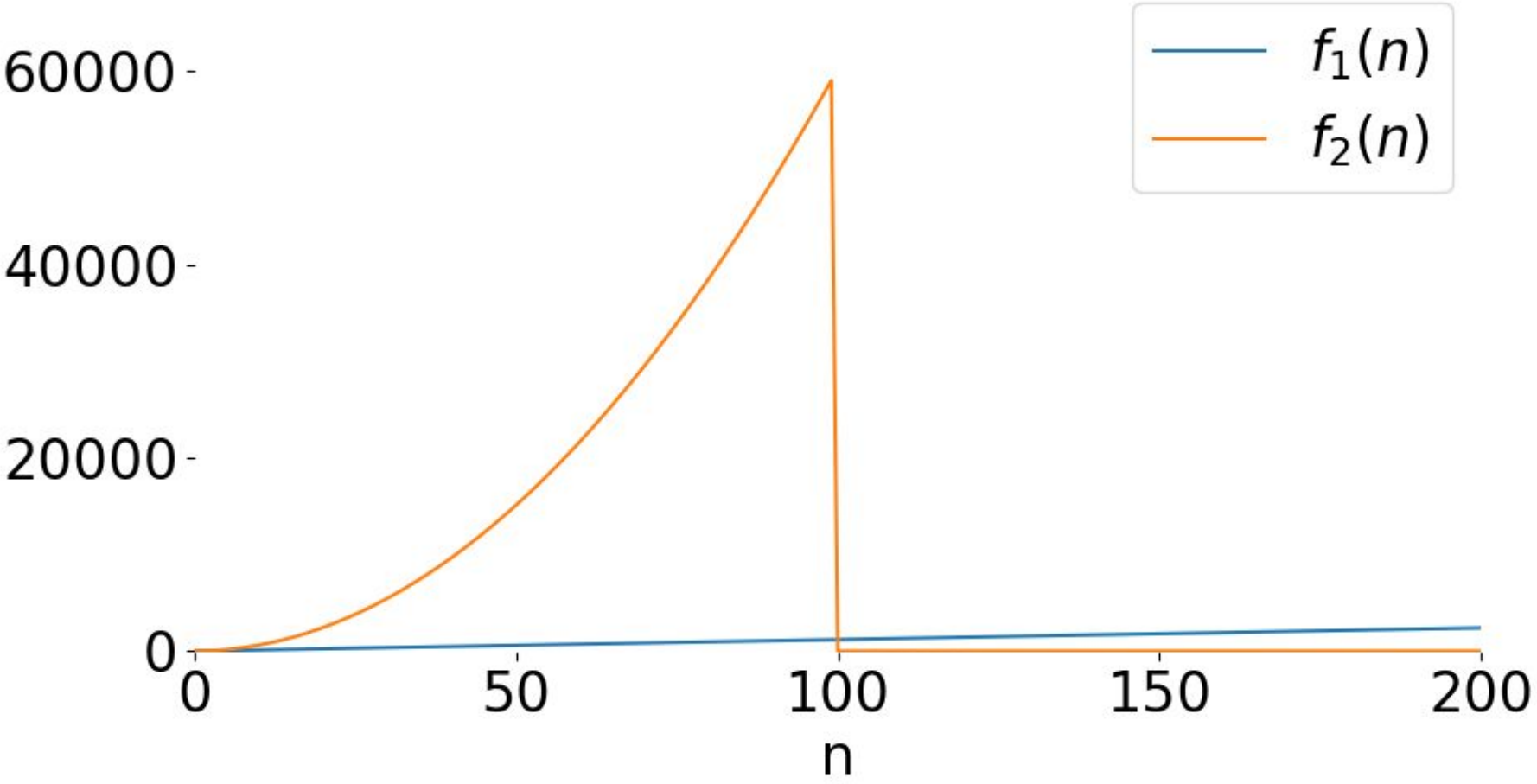
variable access
arithmetic operation
variable assignment

Is Algorithm 1 or Algorithm 2 faster?

$$f_1(n) = 12n + 1$$

$$f_2(n) = \begin{cases} 6n^2 + 2n + 3 & \text{if } n < 100, \\ 6 & \text{otherwise} \end{cases}$$



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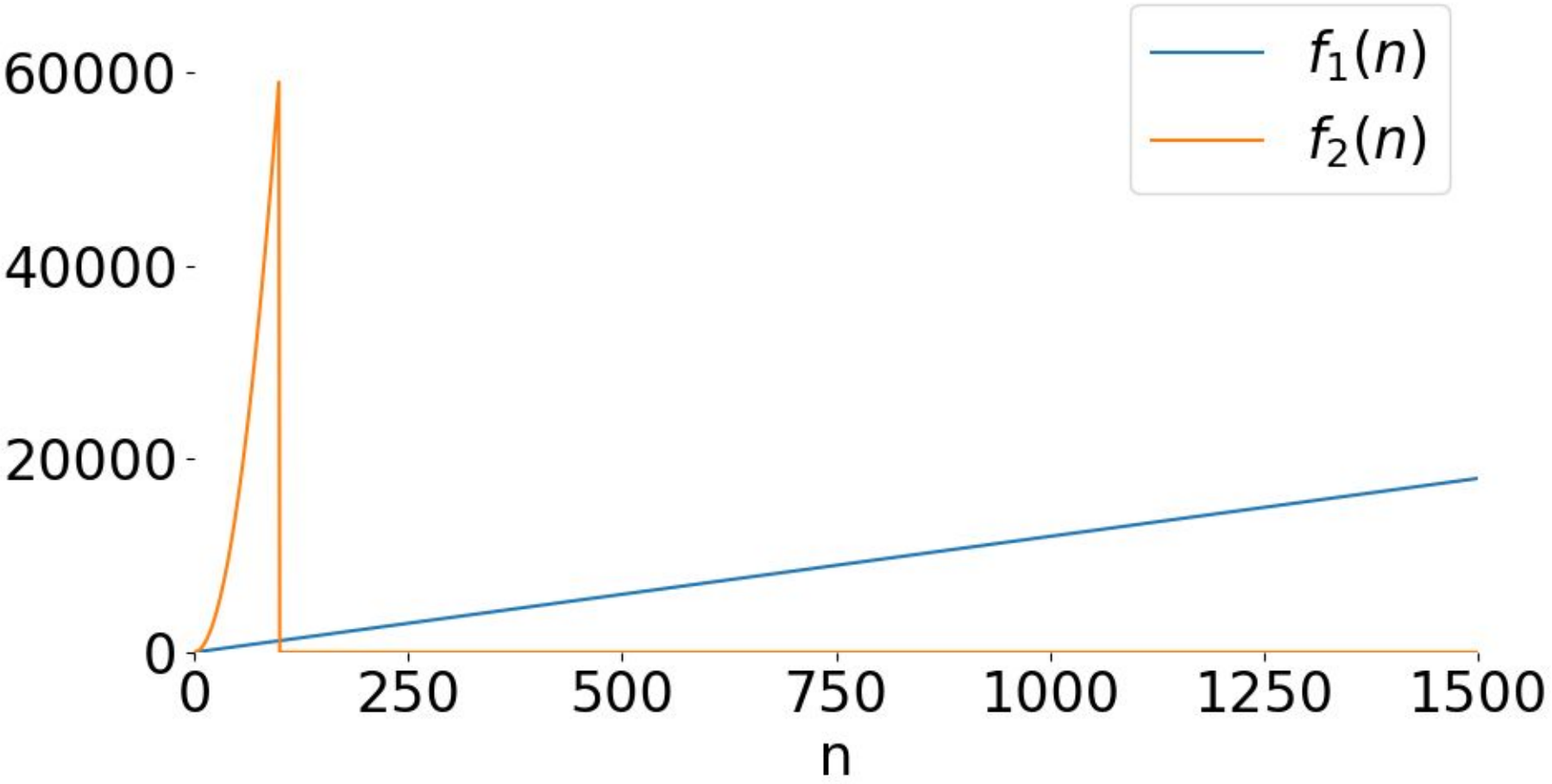

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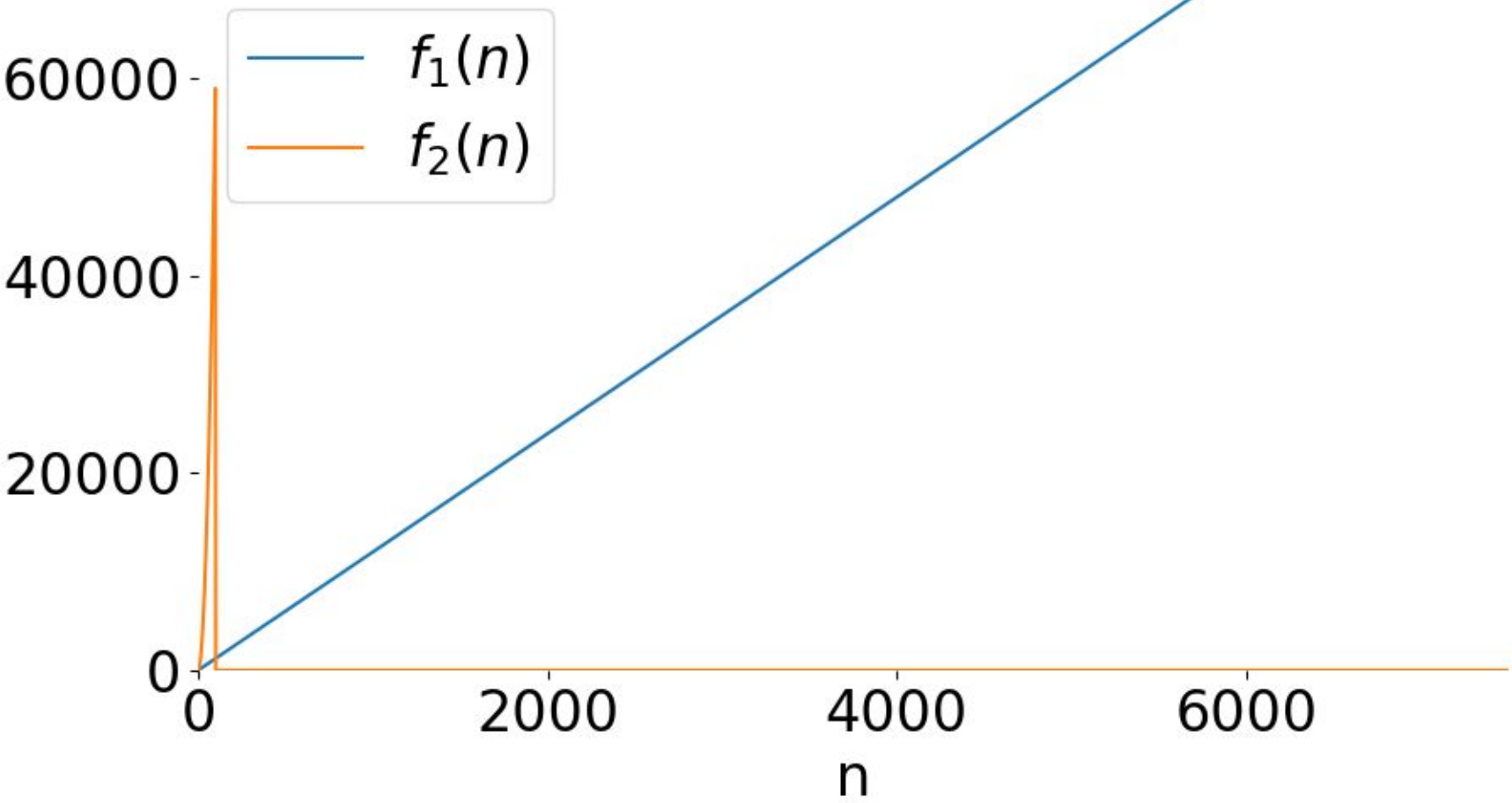


$$f_1(n) = 12n + 1$$

$$f_2(n) = \begin{cases} 6n^2 + 2n + 3 & \text{if } n < 100, \\ 6 & \text{otherwise} \end{cases}$$

<u>Algorithm 3</u>

```
sum = 0
for i = 1 to n:
        sum = sum + 3
```

variable assignment

each loop:
assign i
variable access
arithmetic operation
variable assignment

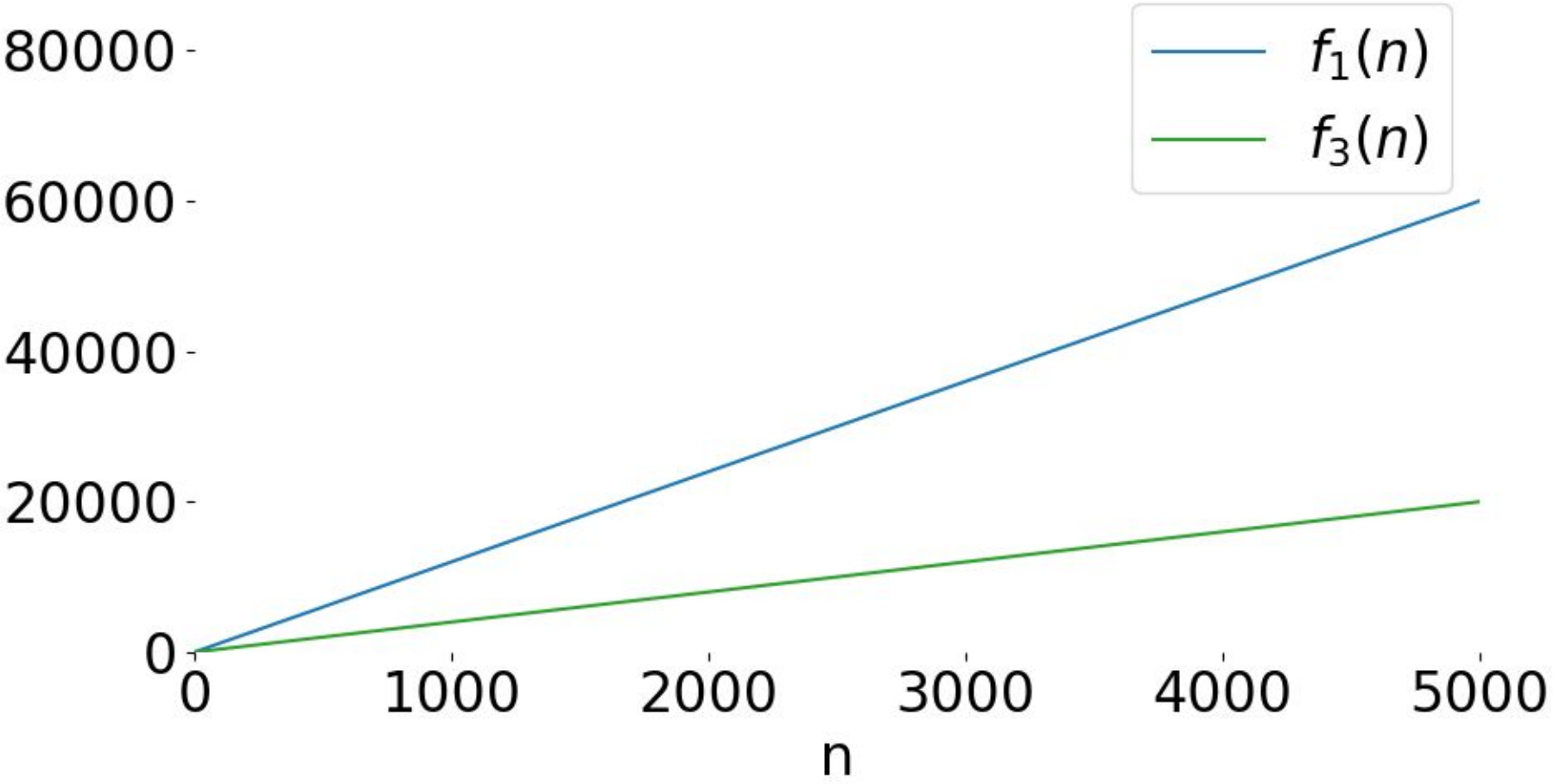
Algorithm 3's runtime:

$f_3(n) = 4n+1$

Is Algorithm 1 or Algorithm 3 faster?

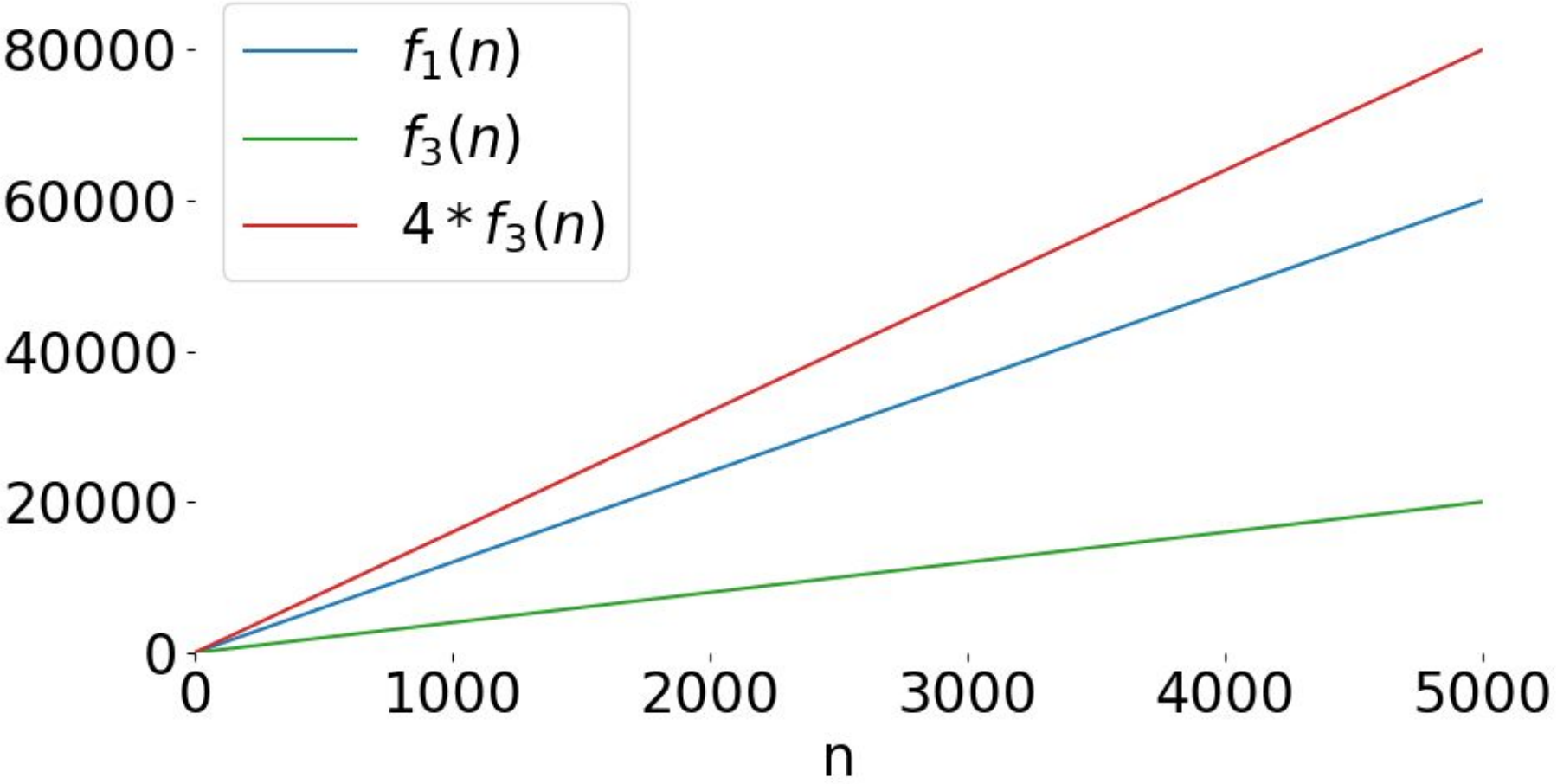
$$f_1(n) = 12n + 1$$

$$f_3(n) = 4n + 1$$



$$f_1(n) = 12n + 1$$

$$f_3(n) = 4n + 1$$



$$f_1(n) = 12n + 1$$

$$f_3(n) = 4n + 1$$