N choose k examples

ex How many different 5-card hands are there when drawn from a 52-card deck?

A clubs, A hearts, J hearts, 2 diam., 4 clubs

we must choose 5 cards without replacement. Order doesn't matter.

So there are

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!(47!)}$$

"52 choose 5"

why is it not $52 \cdot 51 \cdot 50 \cdot 49 \cdot 48$?

what is this number?
it's # of hands if order matters

ex How many different 8-bit strings are there w/ exactly 2 ones?

(recall: there are 2^8 8-bit strings)

eg 11111111
repetition allowed
eg 11111110 diff from 01111111, so order matter

Our 8-bit strings have 8 slots.
We must choose 2 to be ones.

$$\binom{8}{2}$$

ex What is the expected # of aces in a 13-card hand?

Let X = the number of aces in hand.
So we want to compute $E[X]$.

Recall $E[X] = \sum_{y \in \{0,\ldots,43\}} y \cdot Pr[X=y]$.

What is, for example, $Pr[X=0]$?

$$\frac{\#\text{ of ways to get 0 aces}}{\#\text{ of ways to draw 13 cards}}$$

$\#$ of ways to draw 13 cards: $\binom{52}{13}$

$\#$ of ways to get 0 aces: $\binom{48}{13}$

What is $Pr[X=1]$?

$$\frac{\#\text{ of ways to get 1 ace}}{\#\text{ of ways to draw 13 cards}}$$

ways to get 1 ace in 13 card hand:

(suits)

- choose 1 ace, and $\binom{4}{1}$
- choose 12 non-aces $\binom{48}{12}$

Overall, $\binom{4}{1}\binom{48}{12}$

ex what is the probability of drawing a full house?

3 cards same rank

2 cards same rank

2h, 2d, Js, Jh, Jc

$$\frac{\#\text{ of ways to get a full house}}{\#\text{ of ways to draw 5 cards}}$$

$\binom{52}{5}$

choose 1 rank from 13 — choose 3 suits

$\binom{13}{1}\binom{4}{3}$ — group of 3

$\binom{12}{1}\binom{4}{2}$ — group of 2

$\binom{13}{1}\binom{4}{2}\binom{12}{1}\binom{4}{3}$