claim (4.16) If $|x|+|y| \neq |x+y|$ then $xy<0$.

ex

x	y	$\|x\|+\|y\|$	$\|x+y\|$	xy	
-2	3	5	1	-6	T T
2	3	5	5	6	F F

Pf we prove the contrapositive. That is, if $xy \geq 0$ then $|x|+|y| = |x+y|$.
Suppose $xy \geq 0$. WTS $|x|+|y| = |x+y|$.

we prove using cases.

case 1: $x, y \geq 0$.

$|x|+|y| = x+y$ by def of $||$, $x \geq 0$, $y \geq 0$

$x+y = |x+y|$ bc $x,y \geq 0 \Rightarrow x+y \geq 0$, def of $||$

case 2: $x, y \leq 0$.

$|x|+|y| = -x+-y$ def. of $||$, $x,y \leq 0$

$-x-y = -(x+y)$ algebra

$-(x+y) = |x+y|$ bc $x,y \leq 0 \Rightarrow x+y \leq 0$, def of $||$

$\square$

<u>claim</u> All prime numbers are odd.

$\equiv$ If p is prime, then p is odd.

Disproof by counterexample: 2 is prime but 2 is not odd.

claim. Let $p \geq 3$. If p is prime, then p is odd.

<u>Pf</u> We prove the contrapositive. That is, we let $p \geq 3$, and we show that if p is even, then p is not prime.
Let $p \geq 3$ and p even.

p has 2 as a divisor	given
p is not prime	$p \neq 2$, so p has a divisor not equal to itself or 1.

$\square$