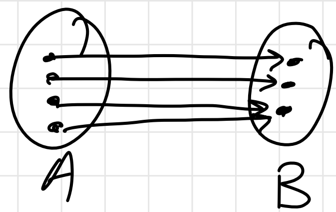
domain codomain

let $f: A \to B$ be a function. → satisfies 3 properties

f onto:

"there exists"

$\forall b \in B : \exists a \in A : f(a) = b$.



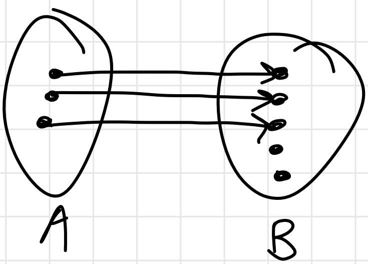
"for all"

f 1:1:

$\forall a_1 \in A, \forall a_2 \in A$

$\forall a_1, a_2 \in A$

$\forall a_1, a_2 \in A : \quad a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$



recall
set builder notation: $\{x \in \mathbb{Z} : 2 | x\}$

"such that"

"for integers m, n, if
m is div. by 3 or n is div. by 3,
then mn is div. by 3"

let $D(x)$ be "x is div. by 3"

$\forall m \in \mathbb{Z}$, $\forall n \in \mathbb{Z}$: $D(m) \vee D(n) \Rightarrow D(mn)$

$\forall b \in B : \exists a \in A : f(a) = b.$

$\forall b \in B : \exists a \in A : f(a) = b$

Note:

let $f : A \to B$ be a function.

$f : A \to B$ is onto $\Rightarrow$ $|A| \geq |B|$

$f : A \to B$ is 1:1 $\Rightarrow$ $|A| \leq |B|$

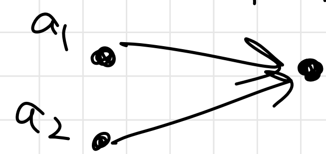
$f : A \to B$ is a bijection $\Rightarrow$ $|A| = |B|$

Theorem 9.13 (Pigeonhole Principle, PHP)

let A, B be sets $f : A \to B$ be a function.

/

If $|A| > |B|$, then there are 2 distinct $a_1, a_2 \in A$ such that $f(a_1) = f(a_2)$.



1) Write a fully quantified statement expressing

2) How does this relate to ?

1) $|A| > |B| \Rightarrow \exists a_1, a_2 \in A : \underbrace{(a_1 \neq a_2) \wedge f(a_1) = f(a_2)}$

$\exists a_1, a_2 \in A, (a_1 \neq a_2) : |A| > |B| \Rightarrow f(a_1) = f(a_2)$

"There exist different a_1, a_2 from A such that if $|A| > |B|$, then $f(a_1) = f(a_2)$"

2) $p \Rightarrow q \qquad \neg q \Rightarrow \neg p$

$\overbrace{f : A \to B \text{ is } 1:1} \Rightarrow \overbrace{|A| \leq |B|}$

$|A| > |B| \Rightarrow \underbrace{f : A \to B \text{ is not } 1:1}$

Proof of PHP:

The PHP is the contrapositive of

$$f : A \to B \text{ is } 1:1 \Rightarrow |A| \le |B| .$$

$\square$

The "pigeon" way of thinking of the PHP:

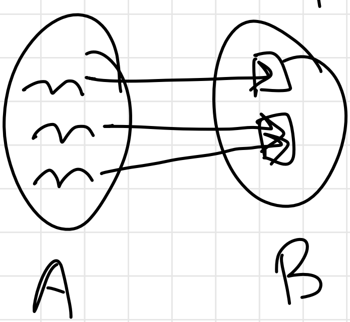
Suppose you have A of pigeons

$|A| = n+1$

B, set of cubbies, $|B| = n$

Each pigeon from A flies into cubby from B.

$f(a) =$ pigeon a's cubby.



≥ 2 pigeons share a cubby.

<u>claim</u> among 13 people, at least 2 share the same birth month.

Proof

Let A (pigeons) be the set of 13 people.

Let B (pigeonholes) be the set of 12 months.

Let $f: A \to B$ $\quad f(a) = a$'s birth month.

Is f a function?

1) each a has a birth month
2) each a has only 1 birth month
3) each birth month is in set of 12 months.

Note that $\left.\begin{array}{l} |A| = 13 \\ |B| = 12 \end{array}\right\} \Rightarrow |A| > |B|$.

Thus, by PHP, $\exists a_1, a_2 \in A$ s.t. $a_1 \neq a_2$ and $f(a_1) = f(a_2)$.

That is, there are 2 distinct people a_1 and a_2 such that their birth months $f(a_1)$ and $f(a_2)$ are the same.