Predicates

So far, our propositions: S is even P

Det A predicate is a Boolean-valued function P: U > Z T, F3 for a set U. That is, a rule or property that a particular entity may or may not have. ex is Even(n) := ST if n is even NEZ ZF if n is odd is Subset (A, B) := ? T if A GB A B sets (F if A ZB $isRat(x) := S T if x \in Q$ $x \in \mathbb{R}$ $Z = F if x \notin Q$

on its own, a predicate P(x) has

no turn value. The value X is unbound. is Even (n) nez can make a proposition by: 1) apply to specific entity let m=5. is Even(m) F is Even (6) T 2) Use quantifiers For all integers n, is Even(n). F T There exists n such that n is even Universal quantifier & "for all" $\forall n \in \mathbb{Z}$: is Even (n) VXES: P(X) "for all x in S, P(X) is true"

T if P(X) evaluates to T for all XES.

Existential quantifier \exists "there exists" $\exists x \in S : P(x)$

"there exists an x in S such that P(x) is $T ext{ if } P(x) ext{ evaluates to T for some } x \in S.$

Examples of propositions w/ quantitiens:

VNEZ: isEven(2n) T

HNEZ: isEven(n2)=7isEven(n) T ∃n∈Z: is Prime(n) T VnEZ: isprime (n) F rationals 3x, y ER: XYEQ AT (XEQ AYER) T not (Neals and

" there exist real numbers x and y such that x times y is rational and not both x and y are rational"

Now to show \exists is true? give one! $x = \sqrt{2}$, $y = \sqrt{2}$. $xy = \sqrt{2}$. $\sqrt{2} = \sqrt{4} = 2$

Precedence rules:

- Y, 3 have highest precedence - () to override or for (larity

<u>ex</u> ¥XES: P(X) => 3yES: P(y)



JXES: (P(X) => JyES: P(y))

All students do not pay full pripon.

let S be the set of all students. (et P(X) be "x pays fil trition" H XES: "P(X)

no student pays full truition.

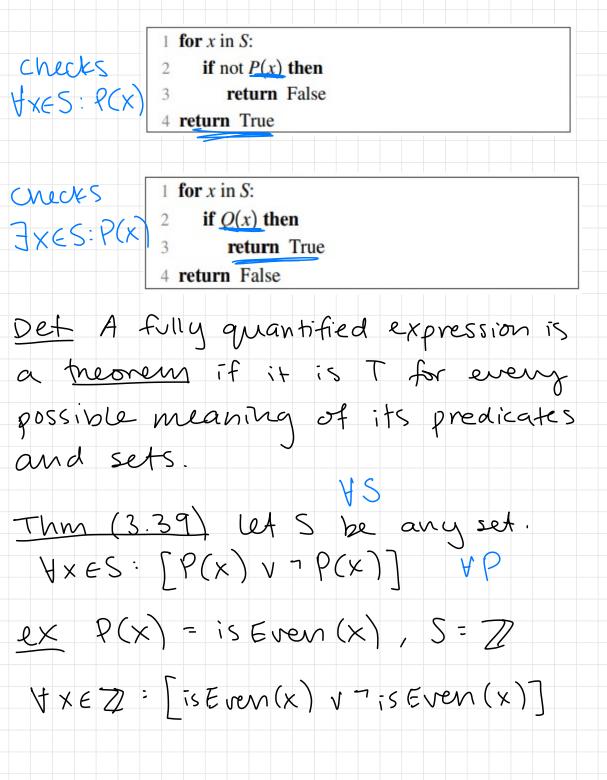
JXES: 7P(X) $7(\forall x \in S : P(x))$

Not all students pay full thit on E

If n² is even, pren n is even.

 $\forall n \in \mathbb{Z}$: is $Even(n^2) = 7$ is Even(n)

Predicales P(X) XEU is Even (X) \checkmark XEZ TF $if \times \%2 == 0:$ do something greater Than 2 (x) uhill (x>2) { do something XER Quantifiers & "for all' universal quantifier VXES: PCX) "for all x in S, P(x) is T" J' there exists " existential quantifier JXES:P(X) " there exists x in S such that P(x) is T"



Non-messeur (3.40) [¥xES:P(x)] V [¥xES:7P(x)]

note: implied US, UP

Disproof that *is* a theorem by counter example.

S = 21,23 P= isEven(X) F [\{ x \in \{ 1, 2\} : is Even (x)] v [\{ x \in \{ 1, 2\} : 7; s Even (x)] consider x = 1 Consider x=2

Det Fully quantified expressions T, Y are logically equivalent (f=Y, fc=>Y) if "P(=> Y" is a meaner.

Theorem (3.41)

7 [4× ES: P(x)]<=>[]×ES: 7P(x)] $\varphi = \Psi$

 $\frac{\text{Theorem}}{2 \times \text{ES}} (3.42)$ $2 \times \text{ES} (x) < 2 \times \text{ES} (x)$

Theorem $\forall x \in \emptyset : P(x) "P(x) is vacuous-$ ly true"