Last time:

- proofs
- direct proofs
- disproof by counter example

<u>Def</u> Let $n, m$ be integers. $n$ is <u>divisible by $m$</u>, if there exists int. $k$ such that $n = mk$

<u>ex</u>

Is 10 divisible by 5?

yes, because $\underset{n}{10} = \underset{m}{5} \cdot \underset{k}{2}$

Is 11 divisible by 5?

$n = 11$, $m = 5$. $11 = 5 \cdot k$ ← 2.2 not integer!

no, because there is no $k$ such that $11 = 5k$

Is $-5$ divisible by 5?

Choose $k = -1$? $-5 = 5 \cdot (-1)$

Is 0 divisible by 2? yes!

Choose $k = 0$. $\quad \underset{n}{0} = \underset{m}{2} \cdot \underset{k}{0}$

**Def** If $n$ is divisible by $m$, we also say $m$ <u>divides</u> $n$.

$m \mid n$ "m divides n"

$2 \mid 10$ but $10 \nmid 2$

"2 divides 10" "10 does not divide 2"

( $x = y$ $\quad x \neq y$ )

"x equals y" "x does not equal y"

(4.12)

**Claim** Let $n$ be an integer. Then $\underline{n \cdot (n+1)^2}$ is even.

Step 1: understand claim

terms: even : divisible by 2

if $n$ is divisible by 2,

then $n = 2k$ for an integer $k$

why is 4 even? $\quad 4 = 2 \cdot \underset{k}{2}$

why is 10 even? $10 = 2 \cdot \underset{k}{5}$

step 2: exampces

| $n$ | $n(n+1)^2$ | $n(n+1)^2$ is even? |
|---|---|---|
| 0 | $0(1)^2 = 0$ | T |
| 3 | $3(4)^2 = 3 \cdot 16 = 48$ | T |
| $-2$ | $-2(-1)^2 = -2$ | T |

we ~~know~~: even # times anything is even.

easy special case:

n is even.

n is odd. then n+1 is even.

**Proof** Consider 2 cases.

Case 1: n is even.

| statements | reasoning |
|---|---|
| $n = 2c$ ~~for~~ int. $c$ | by def. of even |
| $n(n+1)^2 = 2\boxed{c(n+1)^2}$ | by substitution |
| $c(n+1)^2$ is an int. | sum, product of ints ~~is~~ is int |

$n(n+1)^2$ is even

we gave a way to write it as $2k$ for integer $k$

$k$ is $c(n+1)^2$

case 2: $n$ is odd.

| statement | reasoning |
|---|---|
| $n+1$ is even | $n$ is odd |
| $n+1 = 2c$ for integer $c$ | def. of even |
| $n(n+1)^2 = n(2c)^2$ <br> $= n4c^2$ | by substitution, algebra |
| $2nc^2$ is integer | because product of integers is int |
| $n(n+1)^2$ is even | def. of even ($2nc^2 = k$) |

Since $n$ is either even or odd, and in either case $n(n+1)^2$ is even, the claim holds.

$\square$

$$n(n+1)^2 = 4nc^2$$
$$= 2(2nc^2)$$

want:

$n(n+1)^2 = 2k$, $k$ int

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## Proof by cases:

- splits claim into cases
- proves claim in each case
- argues that cases are exhaustive

(example 4.13 in book)

<u>Claim</u> Let $x$ be a real number. Then $-|x| \leq x \leq |x|$.

terms: absolute value $|x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x \geq 0 \end{cases}$

examples:

| | | | | |
|---|---|---|---|---|
| | $x$ | $\|x\|$ | $-\|x\|$ | $-\|x\| \leq x \leq \|x\|$ ? |
| | $3$ | $3$ | $-3$ | $-3 \leq 3 \leq 3$ ? |
| | $-\pi$ | $\pi$ | $-\pi$ | $-\pi \leq -\pi \leq \pi$ ? T T |

<u>Proof</u> Let $x$ be a real number. We prove the claim using cases.

Case 1: $x \geq 0$.

→ $-x \leq x \leq x$ because $x \geq 0$

$|x| = x$ by def. of abs. value, $x \geq 0$

$-|x| = -x$ by algebra

$-|x| \leq x \leq |x|$ substitution

case 2: $x \leq 0$

$x \leq x \leq -x$ because $x \leq 0$

$|x| = -x$ because $x \leq 0$, def. of $||$

$-|x| \leq x \leq |x|$

Because all real numbers are either $\geq 0$ or $\leq 0$ (or both), and the claim holds in either case, the overall claim is true.

case 1: $x > 0$
case 2: $x < 0$
case 3: $x = 0$

**Discrete Structures (CSCI 246)**
in-class activity

Names: ____________________________________________

Please write down the names of everyone in your group above so that you all get credit for today's in-class activity. You should turn in this paper at the end of class.

In this activity, you will begin to prove the following:

**Claim 1.** *Suppose $x$ and $y$ are real numbers. If $|x| \leq |y|$, then $\frac{|x+y|}{2} \leq |y|$.*

1. Are there any terms that your group needs to define to understand the claim? If yes, define them here, consulting another group, an instructor, the textbook, Google, or generative AI if needed:

2. Does your group believe that this claim is true? A yes or no is good enough.

3. What is the converse of the claim?

4. Give three more examples of $x$'s and $y$'s and check whether the claim holds for those examples (that is, fill in three more rows of the table).

| $x$ | $y$ | $\|x\| \leq \|y\|$? | $\frac{\|x+y\|}{2} \leq \|y\|$? | claim holds? |
|---|---|---|---|---|
| 1 | -5 | yes | yes | yes |
| | | | | |
| | | | | |
| | | | | |

claim: if $|x| \le |y|$ then $\frac{|x+y|}{2} \le |y|$

5. Give a proof of the claim **in the case that** $x \geq 0$ **and** $y \geq 0$. You may want to do some scratch work before writing the proof. Use extra paper if needed.

*Proof.* Begin by assuming that $x, y \geq 0$, $|x| \le |y|$.

| statement | reasoning |
| --- | --- |
| $\|x\| = x$, $\|y\| = y$ | def. of abs. val |
| $\|x\| \le \|y\|$ | by assumption |
| $x \le y$ | by substitution |
| $x+y \le y+y$ | adding y to both sides |
| $x+y \le 2y$ | |
| $x+y \geq 0$ | $x, y \geq 0$ |
| $\|x+y\| = x+y$ | def. of abs. value |
| $\|x+y\| \le 2y$ | |
| $\frac{\|x+y\|}{2} \le y$ | subs. |
| $\frac{\|x+y\|}{2} \le \|y\|$ | subs. |

$$\frac{|x+y|}{2} \le |y|$$

□

6. If you have succeeded in proving the first case, complete the proof using additional cases on a new sheet of paper.