(4.19 in book)

claim: Suppose $15x + 111y = 55057$ for $x, y \in \mathbb{R}$, then at least one of $x, y$ is not an integer.

examples:

| $x$ | $y$ | $x$ int | $y$ int | $\geq 1$ of $x, y$ not integer |
| --- | --- | --- | --- | --- |
| $0$ | $\frac{55057}{111}$ | T | F | T |
| $1$ | $\frac{55042}{111}$ | T | F | T |

False start on proving the claim:

Assume $x, y \in \mathbb{R}$, $15x + 111y = 55057$. WTS at least one of $x, y$ not integer.

| statement | reason |
| --- | --- |
| $y = \frac{55057 - 15x}{111}$ | algebra |
| $y = \frac{55057}{111} - \frac{15x}{111}$ | simplifying |

Proof (by contradiction)

Aiming for a contradiction, suppose that the claim is false. That is, suppose that $x, y \in \mathbb{R}$, $15x + 111y = 55057$, but both $x, y \in \mathbb{Z}$

| statement | reason |
| --- | --- |
| $55057 = 15x + 111y$<br>$x, y \in \mathbb{Z}$ | by assumption |
| $55057 = 3(5x + 37y)$ | factoring |
| $\frac{55057}{3} = 5x + 37y$ | |
| $18352\frac{1}{3} = 5x + 37y$ | |
| $18352\frac{1}{3} = K$<br>$K \in \mathbb{Z}$ | sum, prod of integers is integer |

But this is a contradiction, because $18352\frac{1}{3}$ is not an integer.

Therefore, our initial assumption that $x, y \in \mathbb{Z}$ is false.

So at least one of $x, y$ is non-integer. ∎

(4.18)

Claim Let $n \in \mathbb{Z}$. If $n^2$ is even, then $n$ is even.

$n$ even: $2 \mid n$, $\quad \overbrace{n = 2k \text{ for } k \in \mathbb{Z}}$

$n$ odd: $2 \nmid n$, $\quad n = 2c+1$ for $c \in \mathbb{Z}$

7 odd?
$7 = 2 \cdot 3 + 1$

Proof: We prove by contradiction. Assume that if $n^2$ is even, then $n$ is odd.

| | |
|---|---|
| $n = 2k+1$ for $k \in \mathbb{Z}$ | def. of odd |
| $n^2 = (2k+1)^2$ | squaring both sides |
| $n^2 = 4k^2 + 4k + 1$ | algebra |
| $n^2 = 2(2k^2 + 2k) + 1$ | factoring |
| $n^2 = 2c + 1$, $c \in \mathbb{Z}$ | sums, products of ints are ints |
| $n^2$ is odd | def of odd |

This contradicts the fact that $n^2$ is even. So the assumption that $n$ is odd is false. So $n$ must be even.

$\square$