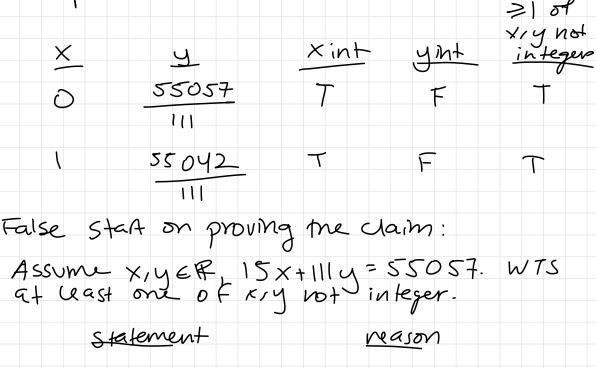
(4.19 in book) claim Suppose 15x + Illy = 55057 for x, y EIR, then at least one of x, y is not an integer.

examples:



 $y = \frac{55057 - 15 \times 111}{111}$ 

algebra

y= <u>55057</u> - <u>15x</u> <u>111</u>

simplifying

troof (by untradiction) Aiming for a contradiction, suppose that the chaim is false. That is, suppose that X, YEIR, ISX + III y = 55057, but both X, yER statement reason 55057=15× +111y x,y eZ by assumption 55057=3(5x+37y) factoring 55057 = 5×137y 18352 = 5×+37y sum, prod of integers is integer  $18352\frac{1}{3} = 4$ KEZ But this is a contradiction, because 18352 ; is not an integer. Therefore, our initial assumption that X, y EZZ is false. So at least one of xiy is non-integer. B

(4.18) Claim let  $h \in \mathbb{Z}$ . If  $n^2$  is even, then n is even.  $n = 2 + fr + E\mathbb{Z}$ 

 $n \text{ odd}: 2 \langle n, n = 2C + 1 \text{ for } C \in \mathbb{Z}$ 7 odd? $7 = 2 \cdot 3 + 1$ 

Proof: we prove by contradiction. Assume that if  $n^2$  is even, then n is odd. n=2k+1 for  $k\in\mathbb{Z}$  def. of odd  $n^2 = (2k+1)^2$  squaring both sides  $n^2 = 4k^2 + 4k + 1$  algebra  $n^2 = 2(2k^2+2k)+1$  factoring  $n^2 = 2C + 1$  soms, products of  $C\in\mathbb{Z}$  ints are ints  $n^2$  is odd det of odd

This contradicts the fact that n2 is even. So the assumption that n is old is false. So n must be even.