Examples of propositions:

- for ints n, $n(n+1)^2$ is even
- for ints n, if n^2 even, then n even
- for $x, y \in \mathbb{R}$, if $x \in \mathbb{Q}$, $y \in \mathbb{Q}$, then $xy \in \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

In proof, we've done:

assume n is even.

$n = 2c$ for $c \in \mathbb{Z}$ ...

$n \in \mathbb{Z}$, $y \in \mathbb{Z}$

$ny \in \mathbb{Z}$

$\sqrt{2}$ rational

$\vdots$

some prop. that is false (contradiction)

We can construct <u>compound propositions</u> out of smaller propositions

Propositions that can't be broken down are <u>atomic</u> propositions.

Syntax vs. Semantics

Syntax → gramatically correct (for a given language)

Semantics → meaning of a gramatically correct statement

$1 \subseteq \mathbb{Z}$ gramatically incorrect → X

$1 \in \mathbb{Z}$ T → 1 is an integer

$1 \notin \mathbb{Z}$ F → 1 is not an integer

Let p, q be propositions.

example: (p = "2 is even", q = "$\sqrt{2}$ is rational")

natural language	syntax in discrete math	informal semantics
p and q	$p \land q$	T iff both p,q T
p or q	$p \lor q$	T iff ≥1 p,q T
not p	$\neg p$	T iff p is F
if p, then q	$p \Rightarrow q$	T iff when p T, q T
p if and only if q	$p \Leftrightarrow q$	T iff p,q match

p exclusive or q $p \oplus q$ T iff p, q mismatch

formal semantics

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

p		q		$p \wedge q$
T	2 is even	T	3 is odd	T
T	2 is even	F	4 is odd	F
F	1 is even	T	3 is odd	F
F	3 is even	F	2 is odd	F

if/then: $p \Rightarrow q$

true if p "forces" q

false if p doesn't "force" q

$p \Rightarrow q$ is false when the promise that p forces q is false

when p is T and q is F   when is this a lie?

ex If $\underbrace{\text{it rains}}_{q}$, then the $\underbrace{\text{grass is wet}}_{q}$.

if rains, then grass wet

p rains	q grass wet	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If p, then q can also be written as:

- p implies q
- p is a sufficient condition for q
- p only if q
- p whenever q
- q is necessary for p

9/15

Common mistakes:

- Setbuilder notation: variable scope

let $A = \{x \in \mathbb{Z} : 3 \mid x\}$ = all ints divisible by 3

$A = \{a \in \mathbb{Z} : 3 \mid a\}$

let $x \in A$
$x \in \{a \in \mathbb{Z} : 3 \mid a\}$

- $\cap$ is not a proposition, it's an operator

$B = \{2, 4, 6\}$, $C = \{2, 4\}$

$A \cap B = \{6\}$ $\qquad$ $A \cap B$

$A \cap C = \emptyset$ $\qquad$ $A \cap C$

$10 - 4$

" S_1, S_2 share at least one element"

$S_1 \cap S_2 \neq \emptyset$

$S_1 \cap S_2$

- reusing variables

<u>Claim</u>: If x, y rational, then xy rational.

<u>Pf</u>: Assume x, y rational.

$x = \frac{n}{d}$, $y = \frac{a}{b}$ — def. of rational

$n, a, d, b \in \mathbb{Z}$, $d, b \neq 0$

$xy = \frac{na}{db}$ — substitution

$xy = \frac{e}{f}$

Review Propositional Logic

$\neg p$	not p
$p \wedge q$	p and q
$p \vee q$	p or q
$p \oplus q$	p exclusive or q
$p \Rightarrow q$	if p, then q (p implies q)
$p \Leftrightarrow q$	p iff q

Def. A <u>truth table</u> lists, for every possible truth assignment, the truth value of a prop.

p	$\neg p$
T	F
F	T

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \wedge q$	$\neg q$	$(p \wedge q) \Rightarrow (\neg q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

Def. 2 propositions are <u>logically equivalent</u> if their truth tables are the same.

p, $\neg\neg p$

$p \equiv \neg\neg p$

$\equiv$: logically equivalent to

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

Def. A proposition is satisfiable if its truth table has at least one T.

Def. A proposition is a <u>tautology</u> if every column of its truth table is T.

p	q	$p \Rightarrow q$
T	T	T
T	F	F ←
F	T	T
F	F	T

For each of:

p, q, 2 diff logical operators

logically equivalent

not satisfiable (no T's)

tautology