

Examples of propositions:

- for ints n , $n(n+1)^2$ is even
- for ints n , if n^2 even, then n even
- for $x, y \in \mathbb{R}$, if $x \in \mathbb{Q}$, $y \in \mathbb{Q}$, then $xy \in \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

In proof, we've done:

assume n is even.

$$n = 2c \text{ for } c \in \mathbb{Z} \quad \dots$$

$$n \in \mathbb{Z}, y \in \mathbb{Z}$$

$$ny \in \mathbb{Z}$$

$\sqrt{2}$ rational

\vdots

some prop. that is false (contradiction)

We can construct compound propositions out of smaller propositions

Propositions that can't be broken down are atomic propositions.

Syntax vs. Semantics

↓
 grammatically correct
 (for a given language)

↘ meaning of a grammatically correct statement

$1 \in \mathbb{Z}$ grammatically incorrect \rightarrow X

$1 \in \mathbb{Z}$ T \longrightarrow 1 is an integer

$1 \notin \mathbb{Z}$ F \longrightarrow 1 is not an integer

Let p, q be propositions.

example: (p = "2 is even", q = " $\sqrt{2}$ is rational")

natural language

syntax in discrete math

informal semantics

p and q

$p \wedge q$

\rightarrow T iff both p, q T

p or q

$p \vee q$

T iff $\exists 1 p, q$ T

not p

$\neg p$

T iff p is F

if p , then q

$p \Rightarrow q$

\rightarrow T iff $\forall n \in \mathbb{N} p, q$ T

p if and only if q

$p \Leftrightarrow q$

T iff p, q match

p exclusive or q $p \oplus q$ T iff p, q mismatch

formal Semantics

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

p

T 2 is even
 T 2 is even
 F 1 is even
 F 3 is even

q

T 3 is odd
 F 4 is odd
 T 3 is odd
 F 2 is odd

$p \wedge q$

T
 F
 F
 F

if/then: $p \Rightarrow q$

true if p "forces" q

false if p doesn't "force" q

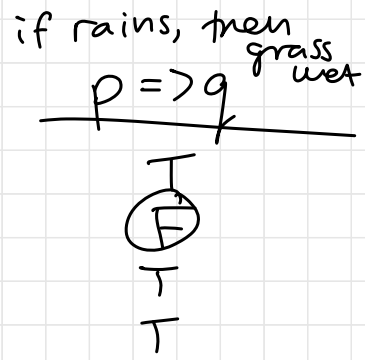
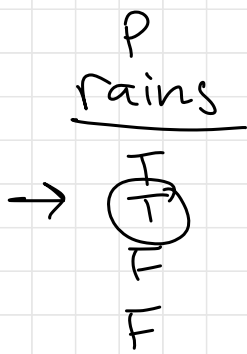
$p \Rightarrow q$ is false when the promise that p forces q is false

when p is T and q is F ↙ ^{man is} this a tie?

ex If it rains, then the grass is wet.

p

q



If p , then q can also be written as:

- p implies q
- p is a sufficient condition for q
- p only if q
- p whenever q
- q is necessary for p

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Common mistakes:

- Setbuilder notation: variable scope

Let $A = \{x \in \mathbb{Z} : 3|x\}$ = all ints divisible
by 3

$$A = \{a \in \mathbb{Z} : 3|a\}$$

$$\text{Let } \left. \begin{array}{l} x \in A \\ x \in \{a \in \mathbb{Z} : 3|a\} \end{array} \right]]$$

- \cap is not a proposition, it's an operator

$$B = \{2, 4, 6\}, \quad C = \{2, 4\}$$

$$A \cap B = \{6\}$$

$$A \cap B$$

$$A \cap C = \emptyset$$

$$A \cap C$$

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" S_1, S_2 share at least one element"

$$S_1 \cap S_2 \neq \emptyset$$

$$S_1 \cap S_2$$

- reusing variables

Claim: If x, y rational, then xy rational.

PF: Assume x, y rational.

$$x = \frac{n}{d}, y = \frac{a}{b}$$

def. of rational

$$n, a, d, b \in \mathbb{Z}, d, b \neq 0$$

$$xy = \frac{na}{db}$$

substitution

$$xy = \frac{e}{f}$$

Review Propositional Logic

<u>$\neg p$</u>	not p
$p \wedge q$	p and q
$p \vee q$	p or q
$p \oplus q$	p exclusive or q
<u>$p \Rightarrow q$</u>	If p , then q (p implies q)
<u>$p \Leftrightarrow q$</u>	p iff q

Def. A truth table lists, for every possible truth assignment, the truth value of a prop.

p	$\neg p$
T	F
F	T

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \wedge q$	$\neg q$	$(p \wedge q) \Rightarrow (\neg q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

Def. 2 propositions are logically equivalent if their truth tables are the same.

$p, \neg\neg p$

$$p \equiv \neg\neg p$$

↑
logically equivalent to

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

Def. A proposition is satisfiable if its truth table has at least one T.

Def. A proposition is a tautology if every column of its truth table is T.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

For each of:

p, q , 2 diff
logical operators

logically equivalent

not satisfiable (no T's)

tautology