Name ______________________________

Problem 1 (25 points)

Consider the relation $\geq$ on integers. That is, for all $x, y \in \mathbb{Z}$, $x \geq y$ if the value of x is greater than or equal to the value of y.

- Is it reflexive?
- Is it irreflexive?
- Is it symmetric?
- Is it anti-symmetric?
- Is it transitive?
- Is it an equivalence relation?
- Is it a partial order?
- Is it a total order?

Problem 2 (25 points)

Complete the proof that $n^3 + 2n$ is divisible by 3 for all nonnegative integers n using mathematical induction by filling in the following. Each blank is worth 1.5 points. The proof of the base case is worth three points. The proof of the inductive case is worth 10 points.

Proof. $n^3 + 2n$ is divisible by 3 for all nonnegative integers n.

First, let $P(n)$ be the predicate that ______________________________________.

We prove that __________________ (something to do with P) using mathematical induction over n.

Base case. We show that _________________ (something to do with P).

Inductive case. We show that ______________________________________ (something to do with P).

Assume _________________ (something to do with P). That is, ______________________________________ (translating the previous blank using the formula, aka the inductive hypothesis, or IH.).

Because we proved both _________________ and ________________________________, by the principle of mathematical induction, $\forall n \geq 0 : P(n)$. $\square$

Problem 3 (25 points)

(a) Let $A = \langle 1, 2, 3, 4, 5 \rangle$ be a 5-element array. How many permutations of A are there?

(b) Suppose we draw a 5-card hand from a standard deck with 52 cards. Define the random variable X to mean the number of aces in the hand. What is $X(\{A\heartsuit, 2\clubsuit, J\clubsuit, Q\diamondsuit, 7\heartsuit\})$?

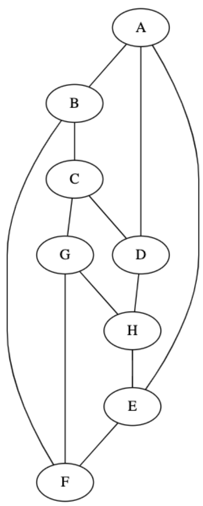
(c) Suppose that the probability you make a free throw is 0.75. Let X be a random variable that is equal to the number of free throws you make in 10 throws. What is $E[X]$?

(d) In a class of 100 people, 60 people prefer dogs over cats, and 40 people prefer pizza over pasta. How many people prefer dogs or pizza? (This may be a range, not a single number.)

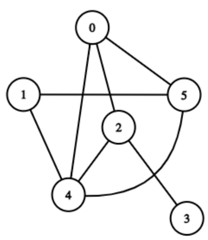
(e) How many different 8-bit strings exist? (For example, 00101110 is an 8-bit string.)

Problem 4 (25 points)

(a) Is the following graph bipartite? (If yes, label the nodes as belonging to L or R.)



(b) Is the following graph planar? (If yes, re-draw it to demonstrate.)



(c) Draw K_4, the complete graph on four nodes.

(d) Draw a directed graph that contains a cycle.

(e) Let S be any set and let R be an equivalence relation on S. Suppose that $a \in S$ is not in the equivalence class of $b \in S$; that is, $a \notin [b]$. Is $(a, b) \in R$, $(a, b) \notin R$, or do we not know?