Discrete Structures (CSCI 246)

Homework 5—bonus

We stated in class, without proof, that big O is transitive: if $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$. Prove that this is true.

Hint: See the proof of Lemma 6.2, Asymptotic Equivalence of Max and Sum, for an example of proof of this type of property.

This isn't graded, but you could think about....

- Since this is a proof that a function is big O of another function, you must show that $f(n)$ and $h(n)$ conform to the definition of $f(n) = O(h(n))$; that is, you must give a $c > 0, n_0 \geq 0 : \forall n \geq n_0 : f(n) \leq c \cdot h(n)$.
- These c, n_0 must be correct.
- Regardless of how you formulate your proof, somewhere you'll need certain facts without which the proof wouldn't work. E.g., if it weren't true that the sum of two integers is an integer, would your proof fail? If so, then that is a fact I need to see stated somewhere.
- The order of these facts must make sense, so that you're not inferring something before you have all the facts to infer it. E.g., you cannot use the fact that the sum of two integers is integer if you don't already know that you have two integers to begin with.
- We need to see a mix of notation and intuition. If you skip too many steps at once, or we cannot follow your proof, or if your proof is overly wordy or confusing, that's not good.