Problem Solving Process

We try to model the following problem solving process during lecture as much as time allows. Of course, watching someone go through the problem solving process is a lot easier than doing it yourself! Give yourself time to try it out; the more you practice with it, the more comfortable you will get. But even the most experienced mathematicians struggle with this; it is part of being a mathematician!

Problem Solving Step #1

Understand what you're trying to do before you try to do it.

- Understand all terminology and all symbols.
- If you don't know what you're trying to do, it is usually impossible to do.

Problem Solving Step #2

Before you try to prove anything, try your proposition out on examples.

- There's no point trying to prove something you don't understand.
- You will likely gain insight & intuition that can help.

Problem Solving Step #3

Draw diagrams! Don't be afraid to use good old-fashioned paper or stand up at a whiteboard. It's amazing what it can do.

- Helps clarify what you're trying to do
- Usually just faster, even when you think it won't be.

Problem Solving Step #4

Consider breaking your proof into multiple cases

- Sure, it takes a bit longer, but
- often it makes your proof easier to write and easier to read.

Problem Solving Step #5

Relatedly, consider trying to focus on one particular case that you think you might know how to prove. For example, maybe you know how to prove the result if the integer is even, or if the two sets are disjoint, etc.

- You can make progress on something,
- Sometimes the proof for the special case can generalize to the more general case, and
- if it can't be generalized, where it breaks can often give insight into how to prove the more general case.

Problem Solving Step #6

Look at propositions/claims/theorems that are similar to yours. Understand how the proof works in that situation, and try to apply the same ideas to your proof.

- Either it works, in which case, great! Or,
- it doesn't work, but where it doesn't work can give you insight into where you need to make changes for your proof to work.

Problem Solving Step #7

If you can't figure out how to prove something true, consider trying to disprove it. Try to come up with a counter-example.

- Even though it is an impossible feat, in trying to construct a counter-example, you will often convince yourself that it is not possible to do. That conviction is the seed of (or a complete) proof!

Problem Solving Step #8

If you're trying to prove some proposition q, it might help to work backwards a few steps from q to help you "recognize" where you're going in the forwards direction. That is, you want your proof to end with

- =>q

So you could ask yourself, what implies q? If r implies q then it would suffice if our proof ended like

- =>r

- =>q

While you probably can't completely go backwards, it can certainly help guide you.

Problem Solving Step #9

Once you have convinced yourself that the proposition is true, it is time to communicate that conviction to your reader in a precise, concise, yet intuitive way. This takes practice!

- Look at the examples from lecture and the textbook to get a sense for the level of detail we require.
- When in doubt, more detail is better than less.
- Every step of a proof should have an explanation as to why you were able to make that step (e.g. "by definition of set intersection").
- Use a mix of precise math definitions and notation, and intuitive word explanations. We need both in a good proof. The definitions and precision ensure we are actually proving the thing we want to prove, but the intuition is necessary for humans to grasp that abstract argument.

Homework Tips

1. Generally the problem sets will be challenging. It is helpful to look at the problems early; even if you don't spend a lot of time on them right away, it helps to have the problems stewing in your head for a few days. Do not try to start the night before it is due.
2. The homework questions will almost always relate to something we have done in class. Therefore, it is a good idea to understand the material from class before spending too much time on the homework. There are also plenty of examples in the textbook. The top of each problem set lists the most relevant lectures where you can and should look for related examples.
3. When you want to prove something, first try to get an intuitive feeling for why it should be true. Try experimenting with examples to see how they fit the mold. Try finding examples that break the mold; often, trying (and failing) to find a counterexample will help you understand why the statement is true in the first place.
4. Your proofs should contain both formal arguments and intuitive explanations. Solutions that consist of notation with no accompanying explanation tend to be basically indecipherable by anyone but the author (and usually indecipherable by the author as well, after a few days pass). Conversely, intuitive explanations without formal proofs are not always sufficient; moreover, our experience is that solutions like this usually turn out to have inaccuracies that render them incorrect.
5. It's in your interest to write up solutions neatly---this not only helps you reread your solution in the future, but it also makes it easier for the grader to understand what you're doing, and to assign partial credit even if it isn't completely correct.
6. You should define any notation that is not commonly used in class. Do not redefine or overload existing notation.
7. Prove any claim that is not obviously true. Each statement should follow by simple reasoning from previous statements.
8. Be patient! Finding proofs is not a process that can be rushed, even by more experienced researchers. Yet another reason to start the whole process as early as possible...
9. I encourage you to work with other students in coming up with ideas for the homework problems. However, you must write up your solutions completely independently. A good rule of thumb when working in groups is that no one should leave with anything written down.
10. Make use of all the class resources: ask questions in lecture, go to office hours, visit the CS help center, read the textbook.

Time-Bound Your Work

When working on something, first try the problem solving process steps as far as you can go without getting stuck.

When/if you get stuck on a part, give yourself 5-20 minutes to put in a good faith effort to figure out how to get unstuck. You might revisit the problem solving process for ideas, such as trying to disprove the claim, or looking for similar results and proofs from lectures.

If you are stuck for 20 minutes, take 5 minutes to formulate a question by stating what you've tried from the problem solving process (which steps did you get through), where you are stuck, and what you have tried to get unstuck. Then either go to office hours with this question, or post it to Discord. Then work on something else until you get a response.

This will keep you more sane. I know that many of us are very stubborn and want to keep working on something until you get it all by yourself, but be kind to yourself. You're new at this! You might get stuck on something early on and that's okay. Rather than spend 3 hours stuck in a hole, someone else can show you a bit of the way out. And the more practice you get with getting out of a hole, the more you'll be able to do it yourself. Remember, the course staff is here to help you not only get a "right" answer, but to give you the tools to be able to construct that "right" answer on your own. It just takes time.

It is not a badge of honor to say "I spent 15 hours on this assignment." You should try to use less time but use that time wisely. Use the course resources.

Of course, doing this requires you to start early enough that your questions are able to be answered. So try to read over the problems and do the first few steps of the problem solving process as soon as possible. You don't have to do all the steps, but at least make sure you understand the terminology and do some examples to get an idea for it.

Specific Proof Tips

Proof Tip #1

To show two sets **A** and **B** are equal (i.e. **A=B**) try breaking it into two smaller & easier proofs

- show **A**⊆**B**, that is, *if* **x** in **A** *then* **x** in **B**
- show **B**⊆**A**, that is, *if* **x** in **B** *then* **x** in **A**

Yes it's 2 proofs, but usually they are correct whereas trying to prove **A=B** in one proof can often be much more difficult.

Proof Tip #2

If the proposition you want to prove says

- “there exists <something> with <some properties>”,
- “there is <something> with <some properties>”,
- or something similar:

Then the first approach to try is to just construct a <something> that has the desired <some properties>.

Proof Tip #3

If you’re doing a proof by contradiction, leave your audience clues!

- Say “By way of contradiction, suppose that...” at the start. Prime the reader!
- Once you derive your contradiction, point it out! say “this is a contradiction to our original assumption, therefore that assumption cannot be true.” It might seem silly to put this in, but it is like putting a thesis statement at the front of each paragraph -- it makes your proofs *so much more readable*.

Tips From the Textbook

textbook link: https://cs.carleton.edu/faculty/dln/book/

4.3.2 - some brief thoughts about proof strategy

4.3.3 - some brief thoughts about writing good proofs

4.5 - common errors in proofs