Direct Proofs and Disproofs by Counter-Example

<u>Def</u> A ~~proposition~~ (statement, claim) is a statement that is either always true or always false. For a proposition, its truth value is its truth or its falsity.

			T/F
<u>ex</u>	Y	$2+2=4$	T
prop??	Y	33 is a prime number	F
	Y	Every integer greater than 2 can be written as the sum of two primes.	??
	Y	$1+2+3+4=10$	T
	n	$1+2+3+4=x$	
	n	Don't forget to do Drill 1!	

In this class, our task is to learn and practice methods of proving proposition T or F.

Def A proof is a convincing argument that a proposition is true. A disproof is an argument that a prop. is false.

claim (from 4.10 in book)

Part 1

Any positive integer n is divisible by 4 if and only if its last two digits are divisible by 4.

part 2

Step 1: understand the proposition.

- what do we mean by "last two digits div. by 4?"

136 (is div. by 4)

3 div. by 4 and 6 div. by 4 ✗

36 is div. by 4 ✓

- what does "n divisible by 4" mean?

there exists an integer k such that $n = 4k$

- positive integer 1, 2, 3, ... not 0, −5, 1.2
- if and only if — Part 1 implies Part 2
  Part 2 implies Part 1 ←

If the last two digits of n are divisible by 4, then n is divisible by 4.

Step 2: do some examples

n	last 2 digits	n div. by 4?	last 2 div. by 4?
20	20	$20 = 4 \cdot 5$ T $n = 4 \cdot k$	T
17	17	F	F
100	(00) 0	$100 = 4 \cdot 25$ T	$0 = 4 \cdot 0$
131	31	N	N

step3: think about special cases that you can already prove.

for example (e.g.), multiples of 100.

<u>Proof</u> let $d_k, d_{k-1}, \cdots, d_1, d_0$ be the digits of n.

$\Rightarrow$ "implies that" $\quad n = d_0 + 10d_1 + \cdots + 10^{k-1}d_{k-1} + 10^k d_k$ because of the def. of base 10

$\Rightarrow \quad n = d_0 + 10d_1 + 100(d_2 + 10d_3 + \cdots + 10^{k-3}d_{k-1} + 10^{k-2}d_k)$

by factoring out 100

$\Rightarrow \quad n = d_0 + 10d_1 + 25 \cdot 4(d_2 + 10d_3 + \cdots + 10^{k-3}d_{k-1} + 10^{k-2}d_k)$

because $100 = 25 \cdot 4$

$$\Rightarrow \quad \frac{n}{4} = \frac{(d_0 + 10d_1)}{4} + \underbrace{\frac{25 \cdot 4(d_2 + 10d_3 + \cdots + 10^{k-3}d_k + 10^{k-2}d_k)}{4}}_{\text{div. by 4}}$$

$\Rightarrow \frac{n}{4}$ is an int. if and only if

$$\frac{(d_0 + 10 d_1)}{4} + \frac{25\,(d_2 + 10 d_3 + \dots + 10^{k-3} d_{k-1} + 10^{k-2} d_k)}{4}$$

is integer

because the two are equal