Warmup problems

Prove or disprove:

e.g. $2^0, 2^1, 2^2, 2^3, \ldots$

1) all odd powers of 2 are negative.

1.5) all odd powers of 2 for exponents greater than 0 are negative. e.g., $2^1, 2^2, 2^3, 2^4, \ldots$

2) All primes are odd.

3) All primes greater than 2 are odd.

Prime: only factors are 1, itself.

10 not prime because 5 is a factor.

1) powers of 2: $\{2^0, 2^1, 2^2, 2^3, \ldots\}$

odd powers of 2: $\{2^0 = 1\}$

F because $\exists$ odd power of 2 that is not negative.

1.5) odd powers of 2 for exponents > 0

$\{\}$ vacuously true

2) 2 is prime.

3) True.

Contradiction. Assume not all primes > 2 are odd. $\exists$ a prime > 2 not odd.

Contrapositive:

claim.

Let $p \in \mathbb{Z}$, $p > 2$.

if p is prime, then p is odd.

if p is even, then p is not prime.

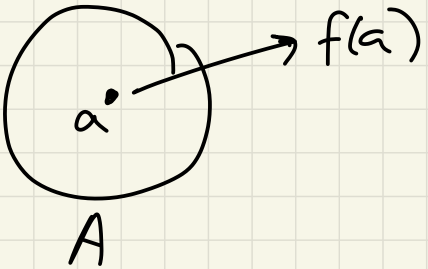
contrapositive

Def Let A, B be sets.

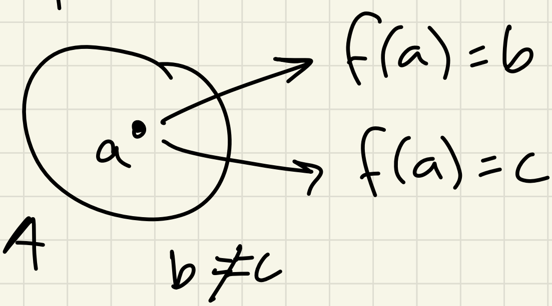
$f: A \to B$ is a <u>function</u> iff it assigns to each $a \in A$ a single value $b \in B$, denoted $f(a)$.

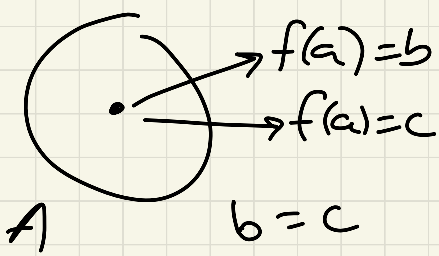
Equivalently, f has the 3 properties:

1) for each $a \in A$, $f(a)$ is defined.



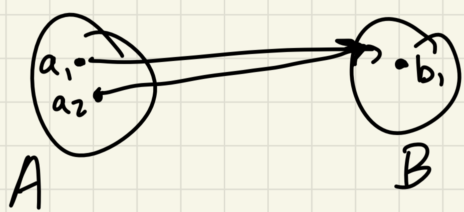
2) for each $a \in A$ $f(a)$ does <u>not</u> produce 2 diff. outputs





NOT a function

3) for each $a \in A$, $f(a) \in B$.



$f : A \to B$

A is the <u>domain</u> of f

B is the <u>codomain</u> of f

The <u>range</u> of f is $\{f(a) : a \in A\}$

$a \in A$	$f(a) \in B$
a_1	$f(a_1)$
a_2	$f(a_2)$
a_3	$f(a_3)$

← some elements of B may have more than one or zero rows

↑ all elements of A have exactly one row

<u>ex</u> $f : \underset{\text{domain}}{\mathbb{R}} \to \underset{\text{codomain}}{\mathbb{R}}$ defined by $f(x) = x^2$.

domain : $\mathbb{R}$
codomain : $\mathbb{R}$
range : $\mathbb{R}^{\geq 0}$

$\mathbb{R}$

intuitive proof of the 3 properties:

$\forall x \in \mathbb{R}$, x^2 is defined (prop. 1)

$\forall x \in \mathbb{R}$, $f(x) = x^2$, a single value (prop. 2)

$\forall x \in \mathbb{R}$, $f(x) \in \mathbb{R}$, because $x^2 \in \mathbb{R}$ (prop. 3) ex let $L : \mathbb{R} \to \mathbb{R}$ defined by $L(x) = \log(x)$.

is L a function? NO!

Prop 1: L defined for all $x \in \mathbb{R}$.

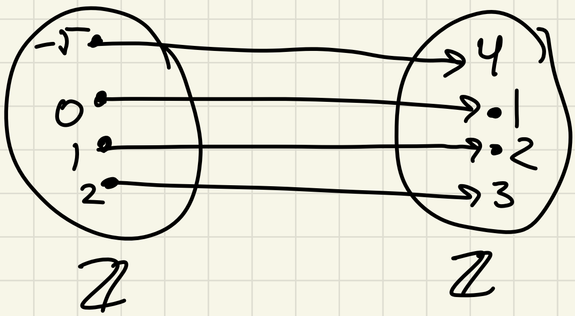
$x = 0$: $\log(0)$ is undefined.

ex $M : \mathbb{R}^{>0} \longrightarrow \mathbb{R}$ defined by $M(x) = \log(x)$.

domain   codomain

ex $s : \mathbb{Z} \to \mathbb{Z}$ def by $s(x) = x + 1$

"successor function"



range : $\mathbb{Z}$