Review

domain ↓ codomain ↙

Let A, B be sets $f: A \to B$ is a function if it satisfies 3 props:

1) $\forall a \in A$ $f(a)$ is defined
2) $\forall a \in A$ $f(a)$ does not produce 2 diff outputs
3) $\forall a \in A$ $f(a) \in B$

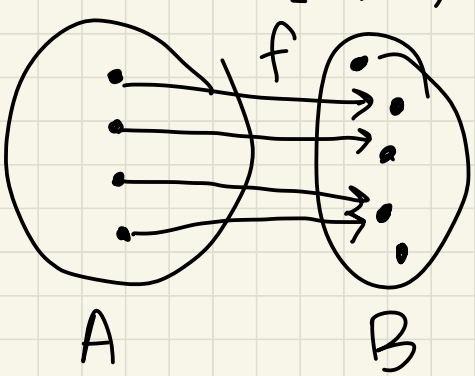
$\{ f(a) : a \in A \}$ range range $\subseteq$ codomain

examples

$f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^2$

$f: \mathbb{R}^2 \to \mathbb{R}$ $f(\langle x, y \rangle) = x$

$E: \mathbb{Z} \to \{T, F\}$ $E(x) =$ is even



$a \in A$	$b \in B$
a_1	$f(a_1)$
a_2	$f(a_2)$
a_3	$f(a_3)$
⋮	

every elt of A has exactly 1 row

doesn't have to have all elts B; dupes okay

ex $S: \mathbb{Z} \to \mathbb{Z}$ defined by $S(x) = x+1$

successor function

domain = $\mathbb{Z}$

codomain = $\mathbb{Z}$

range = $\mathbb{Z}$

claim. $S: \mathbb{Z} \to \mathbb{Z}$ is a function

proof: we prove all 3 properties.

1) $\forall x \in \mathbb{Z}$ $S(x)$ is defined as $x+1$.

2) to show $\forall x \in \mathbb{Z}$, $S(x)$ does not produce 2 diff outputs, we show that if $S(x) = a$ and $S(x) = b$ then $a = b$.

suppose $S(x) = a$ and $S(x) = b$.

$a = x+1$ and $b = x+1$ def. of S

$a = b$ substitution

3) WTS $\forall x \in \mathbb{Z}$, $S(x) \in \mathbb{Z}$.

$S(x) = x+1$, which is an integer because sum of ints is int.

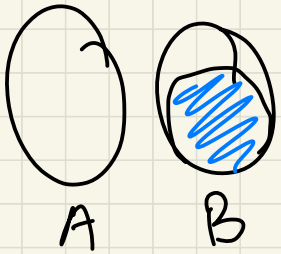
(Notice that $S(x) = x+1$ is a integer $\mathbb{R} \to \mathbb{R}$)

ex $S: \mathbb{R} \to \mathbb{Z}$ $S(x) = x+1$

claim: S is a function

not a function, because violates (3).

let $x = 1.5$. $s(x) = 2.5 \notin \mathbb{Z}$.



recall $f: \mathbb{R} \to \mathbb{R}$ (codomain)

$f(x) = x^2$

range: $\mathbb{R}^{\geq 0}$

ex $f: \mathbb{R}^{\geq 0} \to \mathbb{R}$ defined by $f(x)$ = the number whose absolute value is x

not a function violates prop (2)

$5 \in \mathbb{R}^{\geq 0}$ ✓

$f(5)$ = the number whose abs. val is 5

$5, -5$

(1) find a group 2-5

(2) 1 person get a slip of paper

(3) 15 min to prove that function is/ is not a function

(4) turn in for possible bonus