4.12

<u>Claim</u> Let n be any int. Then $n \cdot (n+1)^2$ is even.

terms: integer ✓
even → div. by 2
→ $x/2$ is an integer

<u>ex</u>

n	$n(n+1)^2$	is $n(n+1)^2$ even?
0	$0(1) = 0$	T
3	$3(3+1)^2 = 48$	T
-2	$-2(-2+1)^2 = -2$	T

easy special cases:

n is even. n times anything is even.
n is odd. so $n+1$ is even.

wait! that covers everything.

<u>Proof</u> Consider two cases.

Case 1: n is even.

<u>statement</u>	<u>reasoning</u>
$n = 2c$ for int c	by def. of even

$n(n+1)^2 = 2c(n+1)^2$	by subs.
$c(n+1)^2$ is an int.	sums, prods of ints are ints
$n(n+1)^2$ is even	we gave a way to write it as $2k$ for int. k (it is $c(n+1)^2$)

Case 2: n is odd.

Statement	reasoning
$n+1$ is even	n is odd
$n+1 = 2c$ for int c	def. of even
$n(n+1)^2 = n(2c)^2$ $= 2n2c^2$	subs., algebra
$n2c^2$ is int.	sums, prods of ints are ints
$n(n+1)^2$ is even	by def. of even

Since n is either even or odd, and in both $n(n+1)^2$ is even, $n(n+1)^2$ is even.

$\square$

Proof by cases: if it is useful, split your claim into cases.
- prove claim in each case
- ensure that the cases are exhaustive (cover all the possibilities)

(4.13)

<u>Claim</u> Let x be a real number. Then $-|x| \le x \le |x|$.

terms: absolute value $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

<u>Proof</u> We prove by cases.

Case 1 : $x \geq 0$. See complete notes 1/23.

Case 2 : $x < 0$.

Because x is either ≥ 0 or < 0, we proved the claim. $\square$