Relations

one CS application: relational databases

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Questions about data stored in relational databases can be posed precisely using the language of relations.

SQL (structured query language) is one language implementing this

Def A <u>binary relation</u> R on sets A, B is a subset $R \subseteq A \times B$.

Recall: $A \times B = \{ \langle a, b \rangle : a \in A, b \in B \}$

We write $(x,y) \in R$ as $x R y$
$(x,y) \notin R$ as $x \not R y$

examples:

① "is (blood) related to" is a binary relation R_1 on people.

Let P be the set of all people.

"is related to" is $\{\langle x,y\rangle : x \in P \wedge y \in P, x \text{ related to } y\}$

$\langle$Serena Williams, Venus Williams$\rangle \in R_1$

$\langle$Lucy Williams, Venus Williams$\rangle \notin R_1$

② $<$ on $A = \{1,2,3,4\}$

$< \; = \{\langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 3,4\rangle\}$

$1 < 2$ but $3 \not< 2$

③ Let $f: A \to B$ be a function

$\{\langle a, f(a)\rangle : a \in A\} \subseteq A \times B$, so it is a relation

<u>Q</u> Is the converse true?

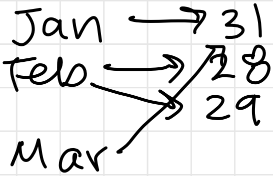
$\{\langle x,y\rangle : x \in A, y \in B\} \overset{?}{\Rightarrow} f: A \to B$ s.t. $f(x) = y$ is a function?

④ Let A = months, B = numbers of days

$\{\langle \text{Jan}, 31\rangle, \langle \text{Feb}, 28\rangle, \langle \text{Feb}, 29\rangle, \langle \text{Mar}, 31\rangle, \ldots\}$

a relation indicating the # days in a month.

Jan	31
Feb	28
Feb	29
Mar	31
Apr	30
⋮	



③ $A = \{1, 2, 3, 4\}$ $\quad 1 \, R_5 \, 4$

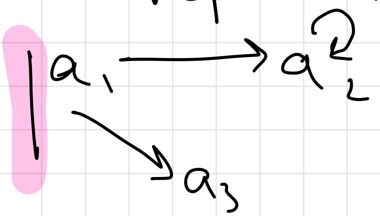
$R_5 = \{ \langle 1,1 \rangle, \langle 1,3 \rangle, \langle 3,1 \rangle, \langle 3,3 \rangle, \langle 2,4 \rangle, \langle 2,2 \rangle, \langle 4,2 \rangle, \langle 4,4 \rangle \}$

⑥ prereq chart

Properties of Relations

Let $R \subseteq A \times A$, so R is a relation on A

Let's represent R as a graph:



$R = \{ \langle a_1, a_2 \rangle, \langle a_2, a_2 \rangle, \langle a_1, a_3 \rangle \}$

R is <u>reflexive</u> if $\forall a \in A : a R a$

all nodes have self-loops

R is <u>irreflexive</u> if $\forall a \in A : a \not R a$

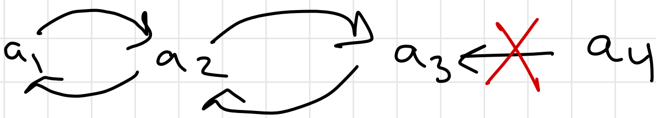
no node has a self-loop

Q are all relations on A either reflexive or irreflexive?

$$\neg\left[\forall a \in A : aRa\right] = \exists a \in A : \neg(aRa)$$
$$= \exists a \in A : a \not R a$$

R is <u>symmetric</u> if

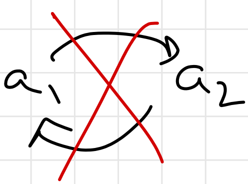
$$\forall a_1, a_2 \in A : a_1 R a_2 \Rightarrow a_2 R a_1$$

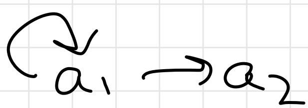


whenever we have a forward edge, we also have a backward edge.

R is <u>anti-symmetric</u> if

$$\forall a_1, a_2 \in A : (a_1 R a_2 \wedge a_2 R a_1) \Rightarrow a_1 = a_2$$

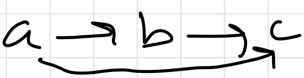




never have backwards edges, but self-loops okay.

R is <u>transitive</u> if

$$\forall a, b, c \in A : (aRb \wedge bRc) \Rightarrow (aRc)$$



shortcut edges always exist.

Q is $a_1 \rightleftarrows a_2$ transitive?

$(a_1 R a_2 \wedge a_2 R a_1) \Rightarrow (a_1 R a_1)$

$$\begin{tikzcd} a_1 \arrow[loop left] \arrow[r, bend left] & a_2 \arrow[l, bend left] \arrow[loop right] \end{tikzcd}$$

ex relation $<$ on $\mathbb{Z}$. $1<2$

- reflexive: $\forall a \in \mathbb{Z}: aRa$ $\quad a<a$?

no - disproof by counterexample. $1 \not< 1$

- irreflexive: $\forall a \in \mathbb{Z}: a \not R a$ $\quad (a \not< a)$

proof: let $a \in \mathbb{Z}$. $a \not< a$. $\square$

- symmetric: $\forall a,b \in \mathbb{Z}: a<b \Rightarrow b<a$.

disproof by counter example: $1<2$ but $2 \not< 1$.

- antisymmetric: