Intro to Graphs

$G = \langle V, E \rangle$

Def An undirected graph $G = (V, E)$ is a non-empty set of vertices / nodes V and a set $E = \{\{u,v\} : u, v \in V\}$ of edges joining pairs of nodes.

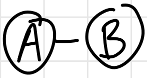
are self-loops allowed?

$\{v\} = \{v, v\}$
$u = v$

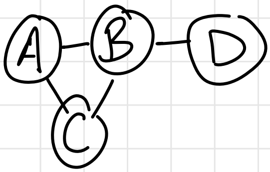
yes!

ex

$V = \{A\}$
$E = \emptyset$



$V = \{A, B\}$
$E = \{\{A,B\}\} = \{\{B,A\}\}$



$V = \{A, B, C, D\}$

$E = \{\{A,B\}, \{A,C\}, \{B,C\}, \{B,D\}\}$

non-ex.

all edges need 2 endpoints

real-world examples

- facebook friends

  nodes: people
  edge: 2 people are friends

- blood relations

Q What property would a relation need to have to be representable as an undirected graph?

ideas:
- irreflexive (no self loops)
- symmetric $a \rightleftarrows b \iff a - b$

A B is this a valid graph?

$V = \{A, B\}$
$E = \emptyset$

Def A <u>directed graph</u> $G = (V, E)$ has a set of vertices and edges $E \subseteq V \times V = \{\langle u, v \rangle : u, v \in V\}$ so that edges are directed from one vertex to another.

Q What else looks like $\subseteq V \times V$? relation!

ex A

$A \to B$ $E = \{\langle A, B \rangle\}$

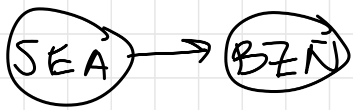
$\neq$

$A \leftarrow B$ $E = \{\langle B, A \rangle\}$

- relations are directed graphs
- functions are also directed graphs

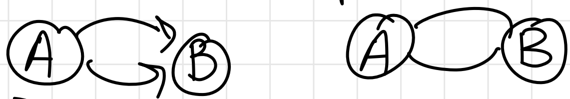
<u>real-world exs</u>

- twitter followers
- transportation networks

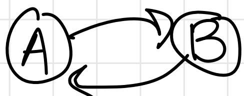


Def A graph is <u>simple</u> if it contains no parallel edges or self-loops.

parallel edges:

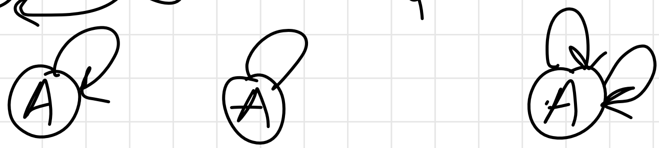


note that



has no parallel edges.

self-loops:



Example 11.3: Self-loops and parallel edges.

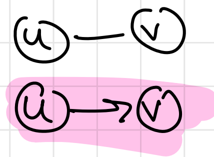
Suppose that we construct a graph to model each of the following phenomena. In which settings do self-loops or parallel edges make sense?

1. A social network: nodes correspond to people; (undirected) edges represent friendships.
2. The web: nodes correspond to web pages; (directed) edges represent links.
3. The flight network for a commercial airline: nodes correspond to airports; (directed) edges denote flights scheduled by the airline in the next month.
4. The email network at a college: nodes correspond to students; there is a (directed) edge $\langle u, v \rangle$ if u has sent at least one email to v within the last year.

	<u>Self-loops</u>	<u>parallel edges</u>
social network	probably no	probably no
the web	yes	yes

flight network | probably no | yes

email network | yes | no



Def let $e = \{u, v\}$ or $\langle u, v \rangle$

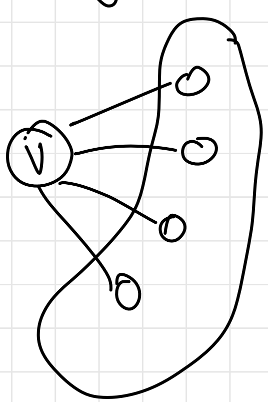
- nodes u, v are <u>adjacent</u> or <u>neighbors</u>
- in a directed graph, v is an <u>out-neighbor</u> of u and u is an <u>in-neighbor</u> of v
- u and v are <u>endpoints</u> of e
- u and v are <u>incident</u> to e ?

(we also say that e is incident to v, u)

Let v be a node in an undirected graph.

$$\text{degree}(v) = \deg(v) = d(v) = \#\text{ of neighbors of } v$$

$$= |\{u \in V : \{v, u\} \in E\}|$$

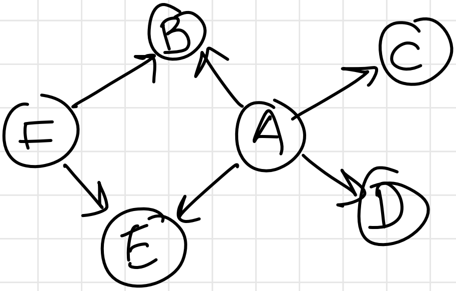


$\deg(v) = 4$

for a directed graph, $\text{indeg}(v) = \#$ of in-neighbors of v

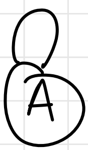
outdeg(v) = # of out-neighbors of v

ex



A, B adjacent
D, C not adjacent
A, B are endpoints of ⟨A, B⟩

F is an *in-neighbor* of B



? ← let's come back to this