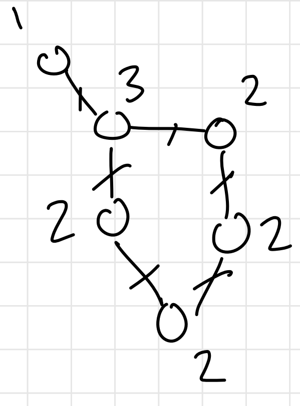
Warmup: label deg. of each node



6 edges, $|E| = 6$

$1 + 3 + 2 + 2 + 2 + 2 = 4 + 8 = 12$

$\sum_{v \in V} \deg(v) = 12$, $2|E| = 2 \cdot 6 = 12$

Theorem 11.8 "Handshaking lemma"

Let $G = (V, E)$ be an undirected graph. Then

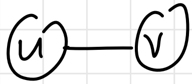
$$\sum_{v \in V} \deg(v) = 2|E|$$

<u>Proof</u> Let $G = (V,E)$ be an undirected graph. Notice that every edge is connected to exactly 2 nodes, meaning that it contributes 1 to the degree of 2 nodes.

So $\sum_{v \in V} \deg(v) = 2|E|$.

More formally, consider this pseudocode.

Algorithm 1 $(G = (V, E))$:



$d_u = 0$ for all $v \in V$
for each edge $\{u, v\}$ in E:
    $d_u = d_u + 1$
    $d_v = d_v + 1$

1. Is the algorithm correct? ✓

2. What is $\sum_{v \in V} d_v$ after iteration i?

$$\sum_{v \in V} d_v = 2i.$$

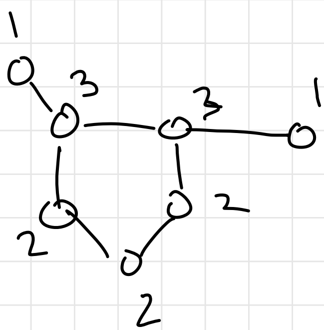
Because the loop runs $|E|$ times,

$$\sum_{v \in V} d_v = 2|E|.$$

fact that follows simply from a previous theorem / lemma

<u>Corollary</u>

Let n_{odd} denote the number of nodes whose degree is odd. Then n_{odd} is even.



Proof Aiming for a contradiction, suppose that n_{odd} is odd.

$$\underbrace{\sum_{v \in V} \deg(v)}_{\text{(1)}} = \underbrace{\sum_{\substack{v \in V: \\ \deg(v) \text{ is odd}}} d_v}_{\text{(2)}} + \underbrace{\sum_{\substack{v \in V: \\ \deg(v) \text{ is even}}} d_v}_{\text{(3)}}$$

(1): $2|E|$, which is even

(2): goal: this is odd. ✓ odd # of odds sums to an odd #

(3): must be even because sum of evens is even

But this contradicts that

$$S = \mathcal{P}(\{0, 1, 2, 3\})$$

$\emptyset$ → $\{1\}$

$\{0, 1, 2\}$

$R_1 = \{ \langle \emptyset, \{1\} \rangle , \langle \qquad \rangle$