Def A path in $G=(V,E)$ is a sequence of nodes $(u_1, u_2, \ldots, u_k)$ s.t.

- $\forall i \in \{1,2,\ldots,k\} : u_i \in V$
- $\forall i \in \{1,2,\ldots,\underline{k-1}\} : \langle u_i, u_{i+1} \rangle$

$u_1 \to u_2 \to u_3 \to \cdots u_{k-1} \to u_k$

Q does this definition allow repeated nodes in a path?

$u_1 = a$
$u_2 = b$
$u_3 = c$
$u_4 = b$
$u_5 = c$
$u_6 = d$

let $U \subseteq V$

ex $\{a, b, c, d\}$ $\qquad \{1,2,3,4,5,6\}$

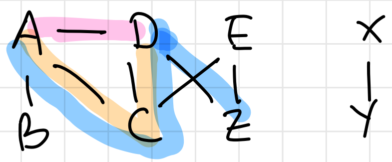
A path is simple if all its nodes are unique.

The length of a path is # edges $(k-1)$.
we say a path traverses its edges.

The shortest path from node u to node v is the path of min. length.

The distance dist(u,v) $d(u,v)$ between u,v is the length of the shortest path.

A graph is connected if $\forall u,v \in V$ $\exists$ path from u to v.

ex 

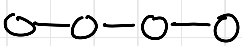
between A, D: AD (shortest), ACD, ACEZD

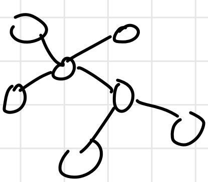
not connected: ~~Z~~ Z to X (path)

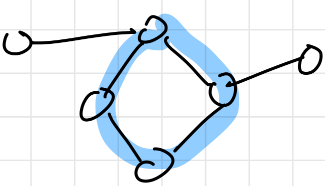
Def A <u>cycle</u> $(u_1, u_2, \dots, u_k, u_1)$ is a path of length ≥ 2 from u_1 to u_2 that does not traverse the same edge twice.

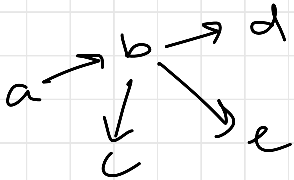
A cycle is <u>simple</u> if its nodes are distinct.

A graph is acyclic if it contains no cycles.

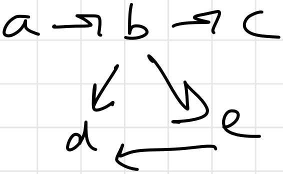
ex  acyclic

 acyclic

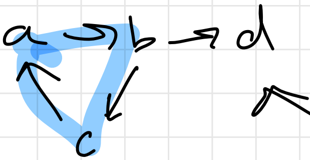
 not acyclic



acyclic



acyclic



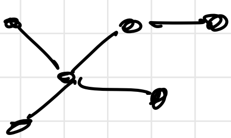
not acyclic

Q How many cycles does have?

$\boxed{1}$ 3 ? ?

We consider $\langle a,b,c\rangle$, $\langle b,c,a\rangle$, $\langle c,a,b\rangle$ equal

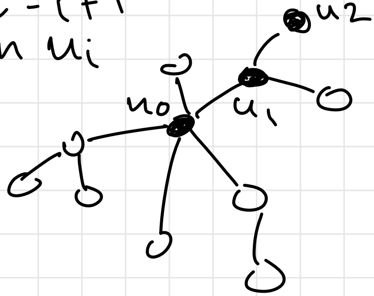
Lemma 11.33 If $G=(V,E)$ is an undirected acyclic graph, then $\exists v \in V$ s.t. $\deg(v)=0$ or $\deg(v)=1$.



Pf We give a proof by construction via an algorithm that, given an acyclic graph, finds a degree 0 or degree 1 node.

alg:

Let u_0 be any node in G
let $i = 0$
while current node u_i has unvisited neighbors:
  let u_{i+1} = such a neighbor
  $i = i+1$
return u_i

(ex 

returns u_2)

Given an acyclic graph G, let t be the node returned by the alg.

WTS either $\deg(t) = 0$ or $\deg(t) = 1$.

Case 1: $t = u_0$ $\quad u_0 \quad$ then $\deg(t) = 0$.

Case 2: $t = u_k$, $k \geq 1$. We will show $\deg(t) = 1$.

Since t is last in sequence $\langle u_0, u_1, \ldots, u_k \rangle$ there is no edge from t to any unvisited node.
If $\exists$ edge from t to any node other than u_{k-1}, it is in $\{u_0, u_1, \ldots, u_{k-2}\}$.

$$u_0 - u_1 - u_2 - \cdots u_j \cdots - u_{k-1} - u_k = t$$

But then $\langle u_j, \ldots, u_k, u_j \rangle$ is a cycle.

So t has only one edge back to u_{k-1}.

$\deg(t) = 1$.