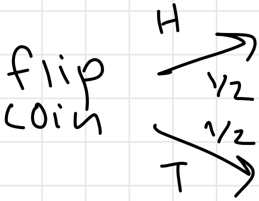
Tree Diagrams in Probability

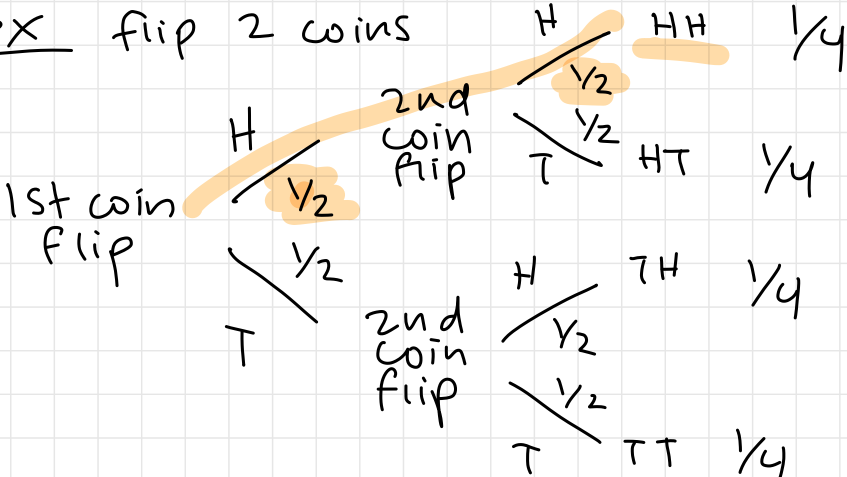
- internal nodes = random choice
  - label w/ probability



sum of probabilities = 1

- leaves are outcomes

ex flip 2 coins

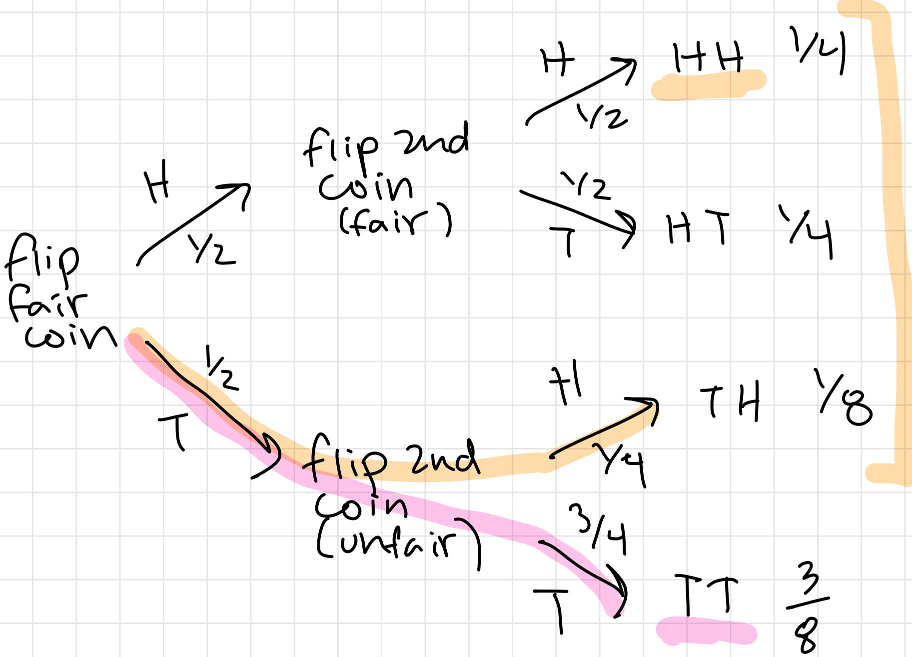


prob. of outcome is product of labels back to root

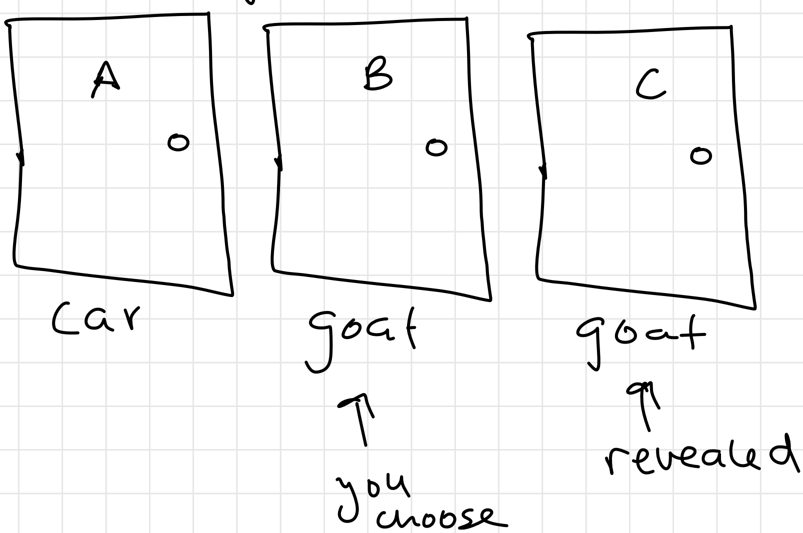
ex flip 1 fair coin. If H, flip 2nd fair coin. If T, flip a coin w/ 0.75 probability of T 0.25 probability of H.

- $Pr[\langle T, T \rangle] = \frac{3}{8}$
- $Pr[\text{at least one } H] = Pr[\overline{\langle T, T \rangle}]$

$1 - \frac{3}{8} = \frac{1}{4} + \frac{1}{4} + \frac{1}{8}$



ex Monty Hall Problem



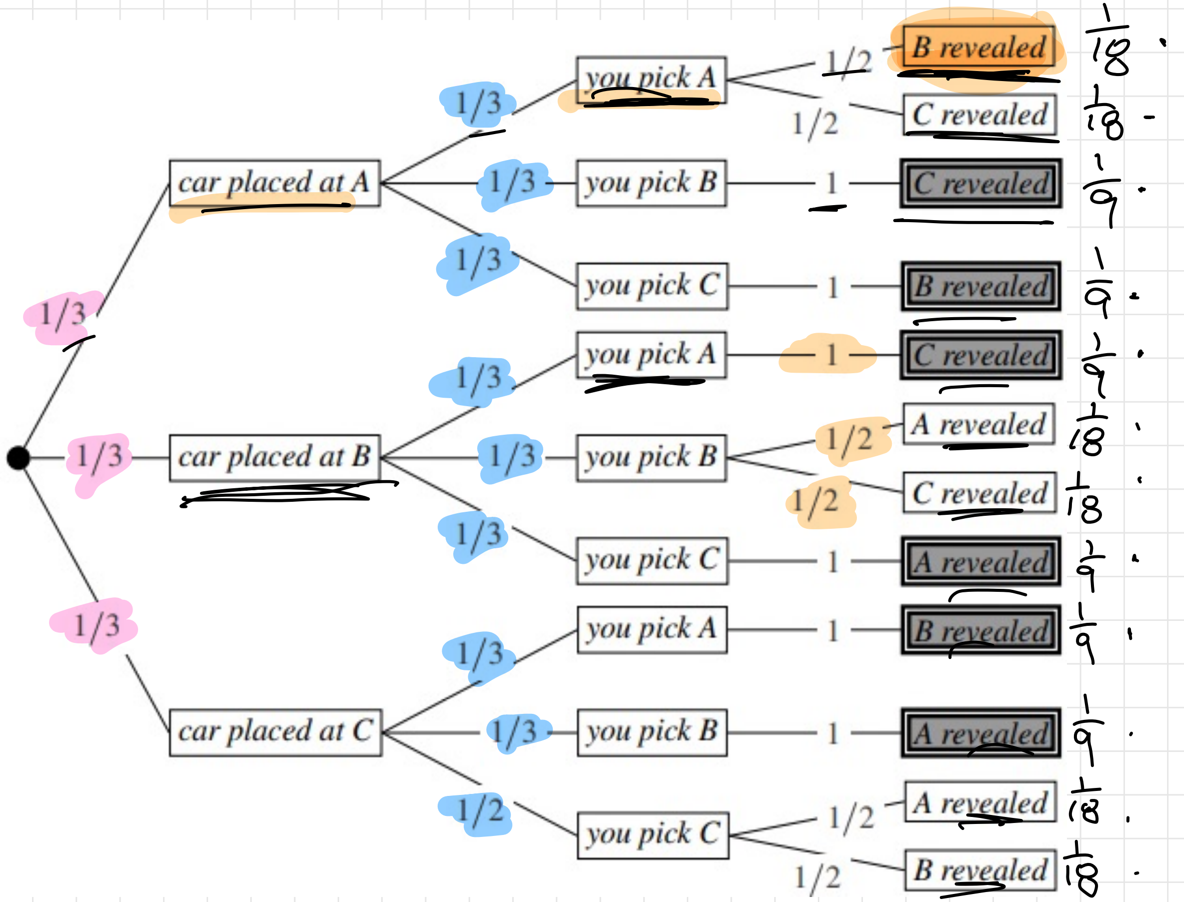
3 doors
2 have goats, 1 has car

You pick a door.

A goat is revealed.

should you switch doors? Yes.

- If you switch, what is your probability of winning?



Let S = set of all outcomes

car at A, you pick A, B revealed $\in S$

Let $A \subseteq S$ be all outcomes where you win by switching.

$\Pr[A]$ = probability of winning when you switch

$$= 6 \cdot \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\Pr[\bar{A}] = 1 - \frac{2}{3} = \frac{1}{3} = 6\left(\frac{1}{18}\right) = \frac{1}{3}$$

Def A *permutation* of a set S is $|S|$ sequence of elements of S with no repetitions.

ex $S = \{1, 2, 3, 4\}$

$\langle 1, 2, 3, 4 \rangle$ ✓
$\langle 2, 3, 4, 1 \rangle$ ✓
$\langle 2, 2, 4, 1 \rangle$ ✗
$\langle 3, 2, 1 \rangle$ ✗

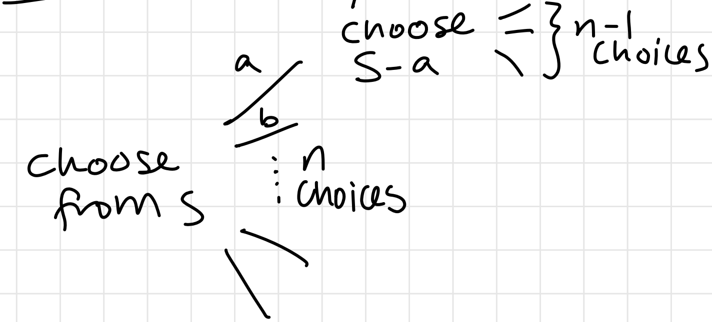
Thm 9.8 Let S be a set w/ $|S| = n$. The number of permutations is $n!$

Proof #1 by product rule. $|A \times B| = |A| \cdot |B|$

Let S_1 be S − first choice, $S_2 = S_1$ − 2nd choice all the way to S_{n-1}

$$|S \times S_1 \times S_2 \times \cdots \times S_{n-1}| = |S| \cdot |S_1| \, |S_2| \cdots |S_{n-1}|$$
$$= n(n-1)(n-2) \cdots (1)$$
$$= n!$$

Proof #2 w/ a tree diagram



$n!$ leaves

Def let n, k be non-negative ints $k \le n$.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

"n choose k"

$\{1, 1, 2\}$

$\{1, 2\}$

the number of ways to choose k elements from a size n set if order doesn't matter

Choosing k items from n

let $S = \{1, 2, 3, 4, 5\}$ $k = 3$

how do I select k items from S?

	repetition allowed	rep. not allowed
order matters	n^k	$\frac{n!}{(n-k)!}$
order doesn't matter	$\binom{n+k-1}{k}$	$\binom{n}{k}$