Def A set is a collection of distinct, unordered items called elements.

ex $D = \{0, 1, 2, 3, \ldots, 9\}$ has 10 element
bits $= \{0, 1\}$ has 2 elements
Bool $= \{\text{True}, \text{False}\}$ has 2 elements
$\mathbb{Z}$ = integers has infinite elements
$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Q}$ = rationals
$\mathbb{R}$ = reals
$V = \{a, e, i, o, u, y\}$ has 6 elts
$\Sigma = \{a, b, c, \ldots, x, y, z\}$ has 26 elts

Def Two sets A and B are equal $(A = B)$ iff A and B contain exactly the same elements.

ex $\{0, 1\} = \{1, 0\} = \{0, 0, 1\}$ size 2

Def We write $x \in S$ $(x \notin S)$ iff x is in S (not in S).

ex $0 \in \text{bits}$ $2 \notin \text{bits}$ $\pi \notin \mathbb{Z}$

Def The cardinality or size of a set S (denoted by $|S|$) is the number of distinct elements of S.

ex $|\text{bits}| = 2$ $|\Sigma| = 26$

Q Can we have a set S such that $|S| = 0$?

yes!

Def The empty set ($\{\}$ or $\emptyset$) is the set with no elements.

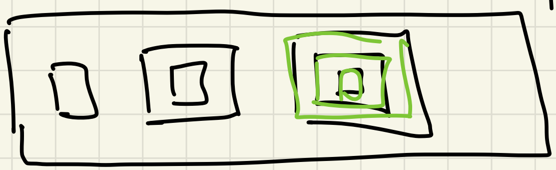
$|\emptyset| = 0$.

Q is $\{\emptyset\} = \emptyset$? No!

$|\{\emptyset\}| = 1$

$F = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$ $|F| = 3$

F is a box with 3 elements



Q If $A = B$ does $|A| = |B|$? yes.

Claim If $|A| = |B|$ then $A = B$ is false.

Disproof by counter example.

$|\{1,2,3\}| = 3$ $|\{4,5,6\}| = 3$

Def Set builder notation defines a set

$$S = \{x : \text{a rule about } x\}$$

S is the set that contains elements x s.t. the rule is true.

ex evens $= \{x : x \in \mathbb{Z} \text{ and } x \text{ even}\}$
$\{x : x = 2c \text{ for } c \in \mathbb{Z}$
$\{x \in \mathbb{Z} : x \text{ is even}\}$

bits $= \{x \in \mathbb{Z} : 0 \le x \le 1\}$

Def A is a subset of B (denoted $A \subseteq B$) iff every element of A is also in B. We can also say that B is a superset of A. ($B \supseteq A$)

ex evens $\subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

$\mathbb{R} \not\subseteq \mathbb{Q}$

$\pi \in \mathbb{R}$ but
$\pi \notin \mathbb{Q}$

bits $\subseteq \{x : x \in \mathbb{Z} \text{ and } 0 \le x \le 9\}$

$\{0, 5\} \not\subseteq \{0, 1\}$

Note $\emptyset \subseteq S$ for any set S
$S \subseteq S$ for any set S