Def A <u>set</u> is a collection of distinct, unordered items called elements.

ex $D = \{0, 1, 2, 3, \ldots, 9\}$ has 10 elements
$\text{bits} = \{0, 1\}$ has 2 elements
$\text{Bool} = \{\text{True}, \text{False}\}$ has 2 elements
$\mathbb{Z}$ = integers has infinite elements
$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Q}$ = rationals
$\mathbb{R}$ = reals
$V = \{a, e, i, o, u, y\}$ has 6 elts
$\Sigma = \{a, b, c, \ldots, x, y, z\}$ has 26 elts

Def Two sets A and B are <u>equal</u> $(A = B)$ iff A and B contain exactly the same elements.

ex $\{0, 1\} = \{1, 0\} = \{0, 0, 1\}$ size 2

Def We write $x \in S$ $(x \notin S)$ iff x is in S (not in S).

ex $0 \in \text{bits}$ $2 \notin \text{bits}$ $\pi \notin \mathbb{Z}$

Def The <u>cardinality</u> or <u>size</u> of a set S (denoted by $|S|$) is the number of distinct elements of S.

ex $|\text{bits}| = 2$ $|\Sigma| = 26$

Q Can we have a set S such that $|S| = 0$?

yes!

<u>Def</u> The <u>empty set</u> ($\{\}$ or $\emptyset$) is the set with no elements.

$|\emptyset| = 0$.

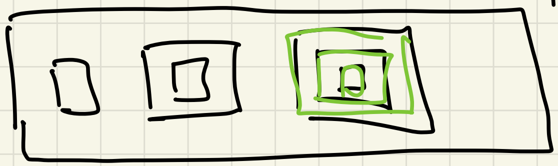
<u>Q</u> is $\{\emptyset\} = \emptyset$? No!

$|\{\emptyset\}| = 1$

$F = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$ $|F| = 3$

F is a box with 3 elements

F



<u>Q</u> If $A = B$ does $|A| = |B|$? yes.

<u>Claim</u> If $|A| = |B|$ then $A = B$ is false.

Disproof by counter example.

$|\{1, 2, 3\}| = 3$ $|\{4, 5, 6\}| = 3$

<u>Def</u> Set builder notation defines a set

$S = \{x : \text{a rule about } x\}$

S is the set that contains elements x s.t. the rule is true.

ex evens $= \{x : x \in \mathbb{Z} \text{ and } x \text{ even}\}$
$\{x : x = 2c \text{ for } c \in \mathbb{Z}$
$\{x \in \mathbb{Z} : x \text{ is even}\}$

bits $= \{x \in \mathbb{Z} : 0 \leq x \leq 1\}$

Def A is a subset of B (denoted $A \subseteq B$) iff every element of A is also in B.
We can also say that B is a superset of A. ($B \supseteq A$)

ex evens $\subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

$\mathbb{R} \not\subseteq \mathbb{Q}$

$\pi \in \mathbb{R}$ but
$\pi \notin \mathbb{Q}$

bits $\subseteq \{x : x \in \mathbb{Z} \text{ and } 0 \leq x \leq 9\}$

$\{0,5\} \not\subseteq \{0,1\}$

Note $\emptyset \subseteq S$ for any set S
$S \subseteq S$ for any set S ←

subset: $\subseteq$ (not $\leq$)

Q If $A \subseteq B$ what can we say about $|A|, |B|$?

$|A| \leq |B|$ If $B \supseteq A$ then $|B| \geq |A|$

converse: If $|A| \leq |B|$ then $A \subseteq B$.

(if a, then b. converse: if b, then a)

If $A \subseteq B$, then $|A| \leq |B|$

Disproof of Converse:

we'll use a counter example.

$A = \{1, 2\}$ $\quad B = \{x, y, z\}$

$|A| = 2$ $\quad |B| = 3$

$A \not\subseteq B$ because $1 \in A$ but $1 \notin B$

<u>claim</u> $\{x \in \mathbb{Z} : 18 | x\} \subseteq \{x \in \mathbb{Z} : 6 | x\}$

The set of numbers divisible by 18 is a subset of the set of numbers divisible by 6.

Every number divisible by 18 is also divisible by 6.

notation: $m | n$ means "m divides n"

"n is divisible by m"

"there exists integer k such that $n = mk$"

<u>ex</u>

x	$18 \mid x$?	$6 \mid x$
18	T	T
6	F	T
0	T	T
1	F	F

Proof WTS $\{x \in \mathbb{Z} : 18 | x\} \subseteq \{x \in \mathbb{Z} : 6 | x\}$

WTS $a \in \{x \in \mathbb{Z} : 18 | x\}$ then $a \in \{x \in \mathbb{Z} : 6 | x\}$

by def. of $\subseteq$.

Suppose that $a \in \{x \in \mathbb{Z} : 18 | x\}$.

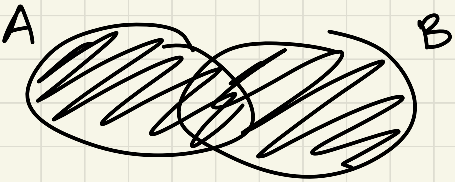
statement	reasoning
$a = 18c$ for $c \in \mathbb{Z}$	by def. of div. by 18
$a = 6 \cdot 3 \cdot c$	by factoring
$a = 6k$ for some $k \in \mathbb{Z}$	because product of ints is int $(3c)$
$6 \mid a$	by def. of div. by 6
$a \in \{x \in \mathbb{Z} : 6 \mid x\}$	rewriting

goal: a is divisible by 6

$a = 6i$ for $i \in \mathbb{Z}$

$0 \in \mathbb{Z}$ $\qquad \{1,2\} \subseteq \mathbb{Z}$ $\qquad \{1,2\} \notin \mathbb{Z}$

Def $A \cup B$ "A union B" is $\{x : x \in A \text{ or } x \in B\}$

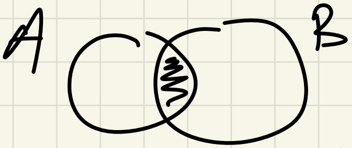


ex $\{2,4,6\} \cup \{2,3,4\} = \{2,3,4,6\}$

evens $\cup$ odds $= \mathbb{Z}$

$A \cup \emptyset = A$ for any set A
$A \cup A = A$

<u>Def</u> $A \cap B$ "A intersect B" $\{x : x \in A \text{ and } x \in B\}$

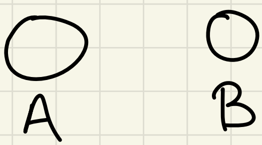


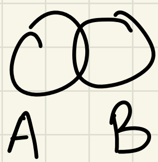
<u>ex</u> $\{2,4,6\} \cap \{2,3,4\} = \{2,4\}$ ← not disjoint

evens $\cap$ odds $= \emptyset$ ← disjoint

$A \cap \emptyset = \emptyset$
$A \cap A = A$

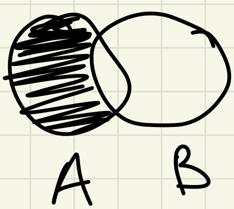
<u>Def</u> Set A, B are <u>disjoint</u> if $A \cap B = \emptyset$.
That is, they have no elements in common.





not disjoint

<u>Def</u> $A - B$ or $A \setminus B$ "A minus B"
$\{x : x \in A \text{ and } x \notin B\}$

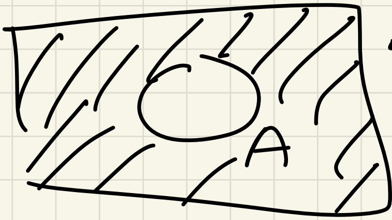


$\{2,4,6\} - \{2,3,4\} = \{6\}$
$\{2,3,4\} - \{2,4,6\} = \{3\}$

evens $-$ odds $=$ evens

$A - B \subseteq A$ for all sets A, B
$A - \emptyset = A$ for all sets A

Def $\bar{A}$ or $\sim A$ "A complement" $\{x : x \notin A\}$



← universe U

$\overline{\{2,4,6\}} = \{\dots -2, -1, 0, 1, 3, 5, 7, 9, 10, 11 \dots\}$
if U is $\mathbb{Z}$

$= \{0, 1, 3, 5, 7, 9\}$
if $\{x : \mathbb{Z} : 0 \le x \le 9\}$ is U

$\overline{\{2,4,6\}}$ if $U = \mathbb{R}$? all reals except 2, 4, 6.

Claim $\overbrace{\{x \in \mathbb{Z} : 2 | x\}}^{A} \cap \underbrace{\{x \in \mathbb{Z} : 9 | x\}}_{B}$
$\subseteq \underbrace{\{x \in \mathbb{Z} : 6 | x\}}_{C}$

If x div. by 2 and x div. by 9, then x is div. by 6.

x	$x \in A \cap B$?	$x \in C$?
6	F	T
0	T	T
3	F	F
18	T	T

<u>Proof</u> If $x \in A \cap B$ then $x \in C$.

Suppose $x \in A \cap B$. WTS $x \in C$.

want to show

goal: show that $x = 6k$ for $k \in \mathbb{Z}$