fact $|P(S)| = 2^{|S|}$

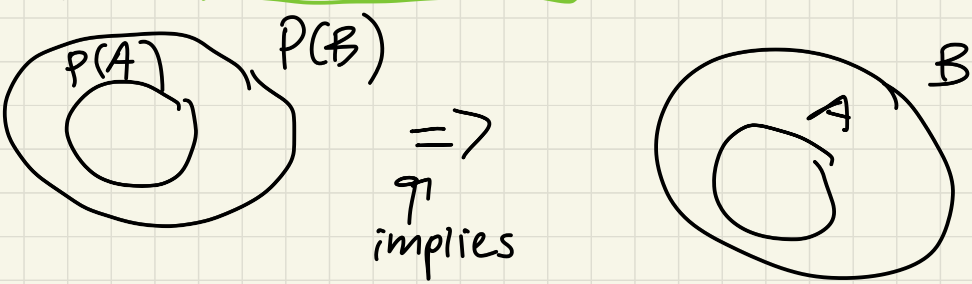
note Power set is also denoted 2^S.

$P(S) = 2^S$ ← set

$\emptyset \in 2^S$ for all S

$S \in 2^S$ for all S

claim if $\boxed{P(A) \subseteq P(B)}$ then $A \subseteq B$.



ex $B = \{1, 2, \{1, 3\}\}$ $P(A) = \{\emptyset, \{1\}\}$

$A = \{1\}$ $|A| = 1$ $|P(A)| = 2^1 = 2$

$\underbrace{P(A) \subseteq P(B)}_{T} \Rightarrow A \subseteq B$

Proof (direct)

Suppose $P(A) \subseteq P(B)$. WTS $A \subseteq B$.

Suppose if $C \in P(A)$ then $C \in P(B)$. WTS if $y \in A$ then $y \in B$.

So we WTS that if $y \in A$ then $y \in B$. we have the fact that if $C \in P(A)$ then $C \in P(B)$ to work with.

Suppose $y \in A$.

$\{y\} \subseteq A$	by def. of $\subseteq$
$\{y\} \in P(A)$	by def. of $P(A)$
$\{y\} \in P(B)$	by $P(A) \subseteq P(B)$ (given)
$y \in B$	by def. of $P(B)$
$A \subseteq B$	by def. of $\subseteq$

Def A sequence/list/tuple/array is an ordered collection of objects.

ex $\langle 0, 1 \rangle$, $\langle 1, 0 \rangle$ not same

$A = \langle a_1, a_2, \ldots, a_n \rangle$ array of n elts

Def Let A, B be sets. the Cartesian Product $A \times B$ is the set of ordered pairs drawn from A and B in that order.

so $A \times B = \{ \langle a, b \rangle : a \in A \text{ and } b \in B \}$

ex $\{a,b,c\} \times \{0,1\} = \{\langle a,0\rangle, \langle a,1\rangle, \langle b,0\rangle, \langle b,1\rangle, \langle c,0\rangle, \langle c,1\rangle\}$

$\mathbb{R} \times \mathbb{R} = \{\langle x,y\rangle : x \in \mathbb{R}, y \in \mathbb{R}\}$ all points in 2D plane

Def For set S, S^n is

$$\underbrace{S \times S \times S \times \cdots \times S}_{n \text{ times}}$$

ex $\mathbb{R}^2$ is $\mathbb{R} \times \mathbb{R}$

$\mathbb{R}^d$ is d-dimensional space

Q $|A \times B| = |A| \cdot |B|$

$|A|, |B|$

Claim $A \times (B \cup C) = \underline{(A \times B) \cup (A \times C)}$

distributive prop: $a(b+c) = ab + ac$

ex $A = \{1,2\}$ $B = \{b\}$ $C = \{\square, 0\}$ $\langle 1,b\rangle = 1b$

$A \times (B \cup C) = A \times \{b, \square, 0\} = \{1b, 1\square, 10, 2b, 2\square, 20\}$

$A \times B = \{1b, 2b\}$

$A \times C = \{1\square, 10, 2\square, 20\}$

$(A \times B) \cup (A \times C) = \{1b, 2b, 1\square, 10, 2\square, 20\}$

<u>Proof</u> we will prove $\subseteq$ and $\supseteq$ separately.

$\subseteq$: Prove that $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$.
That is, if $y \in A \times (B \cup C)$, then $y \in (A \times B) \cup (A \times C)$.

Suppose $y \in A \times (B \cup C)$.

cart. prod

$y \in \langle a, d \rangle$ where $a \in A$ and $d \in (B \cup C)$ — def. of $\times$

There are two cases: either $d \in B$ or $d \in C$.

Case 1: $d \in B$.

$y = \langle a, d \rangle \in A \times B$. — by def. of $\times$

<u>$y \in (A \times B) \cup (A \times C)$</u> — $\cup$ only adds elts. to $A \times B$.