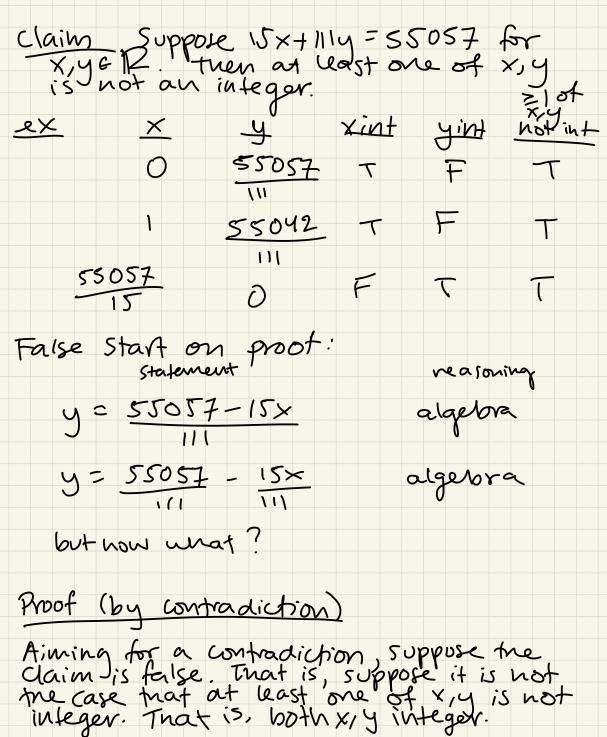
(4.19)



reasoning statement 55057=15x +11/y x,y EZ by claim and 55057 = 3(5×+37y) factoring  $\frac{55057}{2} = 5 \times + 37 \text{ y}$ algebra rewning 18352 = 5x+37y product and sum of ints is int 18352362 But this is a contradiction, because 18352 à is not an integer. Therefore, our initial assumption that both tay 62 is faise. (4.18, but only part) let ne 2. <u>Claim</u> If n<sup>2</sup> is even, men n is even. n n² niseven? n<sup>2</sup> even? × 16 4 T T -2 Y 3 9 Propt For contradiction, suppose the claim is false. That is,

If n² is even then n is odd. Suppose n² is even but n is odd. N=2++1 for KEZ by def. sfodd  $N^2 = (2K+1)^2$ by subs.  $h^2 = 4|c^2 + 4|c + 1$ algebra  $n^2 = 2(2k^2 + 2k) + 1$ factoring C= 2 k 2 + 2 k is integer prod, sum of ints is int  $n^2 = 2C + 1$  for  $C \in \mathbb{Z}$ by def. of odd n<sup>2</sup> isodd This confradicts the fact that n² is even. So the assumption that n is odd is false. (4.20) dain JZ is not rational. Proof For the sake of contradiction, assume that 52 is rational. statement reasoning JZ = J N, d EZ, d = of rapional n, d ave in lowest ferms (don't have a common divisor)

 $2 = \frac{n^2}{d^2}$  $2d^{2} = n^{2}$ n² is even n is even n=2CCEZ $n^{2} = 4c^{2}$  $n^2 = 4c^2 = 2d^2$  $2C^2 = d^2$ d<sup>2</sup> iseven

Squaring both sides

algebra by def. If even by claim 4.18 def. If even squaring both sides

by subs.

algebora def. of even

diseven

by claim 4.18

Ш

So d and h are both div. by 2. But mis contradicts the fact that n,d are in lowest terms. so our initial assumption that Jz is rational is false.