(4.19)

Claim Suppose $15x+111y=55057$ for $x,y \in \mathbb{R}$. Then at least one of x,y is not an integer.

ex	x	y	x int	y int	≥ 1 of x,y not int
	0	$\frac{55057}{111}$	T	F	T
	1	$\frac{55042}{111}$	T	F	T
	$\frac{55057}{15}$	0	F	T	T

False start on proof:

statement	reasoning
$y=\frac{55057-15x}{111}$	algebra
$y=\frac{55057}{111}-\frac{15x}{111}$	algebra

but now what?

Proof (by contradiction)

Aiming for a contradiction, suppose the claim is false. That is, suppose it is not the case that at least one of x,y is not integer. That is, both x,y integer.

statement	reasoning
$55057 = 15x + 111y$, $x, y \in \mathbb{Z}$	by claim and assumption
$55057 = 3(5x + 37y)$	factoring
$\frac{55057}{3} = 5x + 37y$	algebra
$18352\frac{1}{3} = \underline{5x + 37y}$	rewriting
$18352\frac{1}{3} \in \mathbb{Z}$	product and sum of ints is int

But this is a contradiction, because $18352\frac{1}{3}$ is not an integer. Therefore, our initial assumption that both $x, y \in \mathbb{Z}$ is false.

$\square$

(4.18, but only part)

Let $n \in \mathbb{Z}$.

<u>claim</u> If n^2 is even, then n is even.

<u>ex</u>	<u>n</u>	<u>n^2</u>	<u>n is even?</u>	<u>n^2 even?</u>
	4	16	T	T
	-2	4	T	T
	3	9	F	F

<u>Proof</u>

For contradiction, suppose the claim is false. That is, If n^2 is even then n is odd.

$\boxed{\text{Suppose } n^2 \text{ is even}}$ but n is odd.

$n = 2k+1$ for $k \in \mathbb{Z}$	by def. of odd
$n^2 = (2k+1)^2$	by subs.
$n^2 = 4k^2 + 4k + 1$	algebra
$n^2 = 2(2k^2 + 2k) + 1$	factoring
$c = 2k^2 + 2k$ is integer	prod, sum of ints is int
$n^2 = 2c + 1$ for $c \in \mathbb{Z}$	by def. of odd

$\boxed{n^2 \text{ is odd}}$

This contradicts the fact that n^2 is even. So the assumption that n is odd is false.

(4.20)

Claim $\sqrt{2}$ is not rational.

Proof For the sake of contradiction, assume that $\sqrt{2}$ is rational.

statement	reasoning
$\sqrt{2} = \frac{n}{d}$ $n, d \in \mathbb{Z}, d \neq 0$	def. of rational
n, d are in lowest terms (don't have a common divisor)	

$2 = \frac{n^2}{d^2}$	squaring both sides
$2d^2 = n^2$	algebra
n^2 is even	by def. of even
n is even	by claim 4.18
$n = 2c \quad c \in \mathbb{Z}$	def. of even
$n^2 = 4c^2$	squaring both sides
$n^2 = 4c^2 = 2d^2$	by subs.
$2c^2 = d^2$	algebra
d^2 is even	def. of even
d is even	by claim 4.18

So d and n are both div. by 2. But this contradicts the fact that n, d are in lowest terms. So our initial assumption that $\sqrt{2}$ is rational is false.

$\blacksquare$