Examples of propositions:

$\boxed{\text{for ints } n \qquad n(n+1)^2 \text{ is even}}$

for ints n — if n^2 even, then n even

for $x, y \in \mathbb{R}$ — if $x \in \mathbb{Q}$, $y \in \mathbb{Q}$ then $xy \in \mathbb{Q}$

$\sqrt{2}$ is not rational

In proofs, we've done

n even $\Rightarrow$ $n = 2c$ for $c \in \mathbb{Z}$ $\Rightarrow \cdots$
("implies that")

$nx, ny \in \mathbb{Z} \Rightarrow nx\,ny \in \mathbb{Z}$

$\sqrt{2}$ rational $\Rightarrow \cdots \Rightarrow$ false (contradiction)

we can construct compound prop. out of smaller (atomic) prop.

can't be broken down

if $\underbrace{\overbrace{\boxed{\text{n is integer}}}^{p}}$ then $\underbrace{\boxed{n(n+1)^2 \text{ even}}}_{q}$

Syntax vs. Semantics

grammatically correct (for a given language)

meaning of a grammatically correct sentence or statement let p, q be prop.

natural lang	syntax	informal semantics
p and q	$p \land q$	T iff both p, q T
p or q	$p \lor q$	T iff ≥ 1 of p, q T
not p	$\neg p$	T iff p is false
if p then q	$p \Rightarrow q$ ($p \to q$)	T iff when p T, q T
p if and only if q	$p \Leftrightarrow q$	T iff p, q match
p exclusive or q	$p \oplus q$	T iff p, q mismatch

formal semantics (truth table)

p	q	$p \land q$	$p \lor q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

	p			q	$p \land q$
T	2 is even	and	T	3 is odd	T
T	2 is even	and	F	4 is odd	F
F	1 is even	and	T	3 is odd	F
F	3 is even	and	F	2 is odd	F

if / then

true iff p "forces" q (false if p doesn't force q)
it's a promise that whenever p T, q also T
so $p \Rightarrow q$ is F when that promise is broken
That is, when p is T and q is F, $p \Rightarrow q$ is F

ex If it rains then the grass is wet.

when is a lie?

p rains	q grass wet	rain ⇒ grass wet
T	T	T
T	F	F
F	T	T
F	F	T

if p then q can also be written as:

q whenever p
q is necessary for p
p only if q
p is a sufficient condition for q
whenever p also q
p implies q

Def 2 propositions are logically equivalent iff their truth tables are the same

p	¬p	¬¬p
T	F	T
F	T	F

$p \equiv \neg\neg p$

Def prop p is satisfiable iff its truth table has at least one T. That is, it's true under at least one truth assignment.

Def A prop. is a tautology iff every row of the truth table is T

ex $(p \Rightarrow q) \vee p$ scratch work

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \vee p$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Q Suppose we have propositions p, q, r. How many rows does the truth table have?

8. In general, 2^n. One for each of $\{T, F\}^n$.

$$\{T, F\}^n = \underbrace{\{T, F\} \times \{T, F\} \times \cdots \times \{T, F\}}_{n \text{ times}}$$

for $n = 3$,

$$\{T, F\} \times \{T, F\} \times \{T, F\} = \{\langle T, T, T\rangle, \langle T, T, F\rangle, \dots\}$$

A drill question: how many rows does the truth table for $a \Rightarrow (b \vee (c \wedge \neg a))$ have?

$2^3 = 8$.

truth table for

a	b	c	$\neg a$	$c \wedge \neg a$	$b \vee (c \wedge \neg a)$	$a \Rightarrow (b \vee (c \wedge \neg a))$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Precedence Rules

parentheses

1. $\neg$
2. $\vee, \wedge, \oplus$
3. $\Rightarrow$
4. $\Leftrightarrow$

break ties L to R

$c \wedge \boxed{\neg a}$

Def (again) 2 propositions p, q are <u>logically equivalent</u> if their truth tables are the same. We write $p \equiv q$ if this is true.

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

De Morgan's Law: $\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$