Def A predicate is a Boolean-valued function $P: U \to \{T, F\}$ for a set U.
That is, a rule / property that a particular entity may / may not have.

ex $\text{isEven}(n) := \begin{cases} T & \text{if } n \text{ is even} \\ F & \text{if } n \text{ is odd} \end{cases}$

$n \in \mathbb{Z}$ "defined to be"

$\text{isPrime}(n) := \begin{cases} T & \text{if } n \text{ prime} \\ F & \text{if } n \text{ not prime (composite)} \end{cases}$

$n \in \mathbb{Z}^{>0}$

$\text{isSubset}(A, B) := \begin{cases} T & \text{if } A \subseteq B \\ F & \text{if } A \not\subseteq B \end{cases}$

A, B sets

$\text{isRat}(x, y) := \begin{cases} T & \text{if } x/y \in \mathbb{Q} \\ F & \text{if } x/y \notin \mathbb{Q} \end{cases}$

$x, y \in \mathbb{R}$

On its own, a predicate $P(x)$ has no truth value. The value x is unbound.

Can make it a prop. by applying it to a specific entity.

ex $\text{isEven}(n)$ ✗ $\quad$ $\text{isEven}(1)$ ✓ F

$\text{isPrime}(n)$ ✗ $\quad$ $\text{isEven}(2)$ ✓ T

$\text{isPrime}(2)$ ✓ T

Can also use quantifiers:

ex there exists int. n such that isEven(n).

for all int n isEven(n).

there exists int n s.t. isEven(n) and $\neg$isEven(n).

Universal Quantifier $\forall$ "for all"

$\forall x \in S : P(x)$

"for all x in S, P(x) is true"

true iff P(x) evaluates to T for every $x \in S$

Existential Quantifier $\exists$ "there exists"

$\exists x \in S : P(x)$

"there exists x in S such that P(x) is true"

T iff P(x) evaluates to T for some $x \in S$.

These quantifiers bind x.

ex $\forall x \in \mathbb{Z}$: x is even (isEven(x)) F

$\forall x \in \mathbb{Z}$: 2x is even T

$\forall n \in \mathbb{Z}$: If n^2 is even then n even T

$\forall x \in \mathbb{Z}$: if 7x+9 even then x odd T

$\forall x, y \in \mathbb{R}$: if x/y irrational, then x or y irrat. T

x/y irrational $\Rightarrow$ x irrat. $\vee$ y irrat.

$\exists n \in \mathbb{Z} : n$ is not even T $(n=3)$

$\exists n \in \mathbb{Z} : n$ is prime T $(n=3)$

$\exists n \in \mathbb{Z} : n$ is even and n is odd F

$\exists x, y \in \mathbb{R} : x \cdot y \in \mathbb{Q} \wedge \neg(x, y \in \mathbb{Q})$ T

$x = \sqrt{2}$, $y = \frac{1}{\sqrt{2}}$

Precedence

- $\forall, \exists$ highest precedence
- () to override or for clarity

ex $(\forall x \in S : P(x)) \Rightarrow (\exists y \in S : P(y))$ thm

$\forall x \in S : [P(x) \Rightarrow \exists y \in S : P(y)]$ non-thm

We have to be careful when going btwn English language and quantified statements.

ex All students do not pay full tuition.

an attempt: $\forall$ students x ($x \in$ students) : x does not pay full tuition.

but this means that no student pays full tuition.

"not all students pay full tuition"

$\neg(\forall$ students x : x pays full tuition$)$

$\exists$ a student x : x does not pay full tuition

ex All ints are not even $\forall x \in \mathbb{Z} : \neg(\text{isEven}(x))$
Not all ints are even $\neg(\forall x \in \mathbb{Z} : \text{isEven}(x))$

≥ 1 ints x, y are even
$\equiv \text{isEven}(x) \vee \text{isEven}(y)$

x is even but y is odd
$\equiv \text{isEven}(x) \wedge \neg \text{isEven}(y)$

unbound variables

ex n^2 even $\Rightarrow n$ even
$\forall n \in \mathbb{Z}$

assume $\forall$