This document contains explanations of notation and definitions that we may go over quickly in lecture. If you were confused about notation or think you missed a definition, look here!

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- <u>Def</u> An integer n is <u>divisible</u> by integer m if there exists an integer k such that $n = mk$.

We sometimes say "m divides n" to mean the same thing as "n is divisible by m."

We use the shorthand $m|n$ to say m divides n.

Another equivalent definition of divisibility is that $m|n$ if and only if $\frac{n}{m}$ is an integer.

<u>ex</u> $\overset{n}{0}$ is divisible by $\overset{m}{2}$ because we can choose $k=0$ and write $0 = 2 \cdot 0$.

5 is not divisible by 4 because there is no integer k so that $5 = 4k$.

-33 is divisible b 11 because $-33 = 11 \cdot (-3)$.

- The ellipsis (…) notation in math:

… means "continuing onward in the same manner."

So 1, 2, …, 99, 100 means "all of the integers between 1 and 100."

By convention, we put two items at the start (here, 1 and 2) and two at the end to be very explicit about the pattern. But in general, look at examples and use your own judgment about how to use … .

ex $-100, -98, \ldots, -4, -2$ even negative integers between -100 and -2

$\ldots -2, -1, 0, 1, 2, \ldots$ all integers

$c_0 x_0 + c_1 x_1 + c_2 x_2$ polynomials up to degree 2

$c_0 x_0 + c_1 x_1 + \cdots + c_{k-1} x_{k-1} + c_k x_k$ polynomials of degree k

- Exponent math rules.

We can simplify expressions with exponents as long as they share the same base!

$$5^{8 \leftarrow \text{exponent}}$$

base

<u>ex</u> 5^8 divided by 5^2 is $\frac{5^8}{5^2} = 5^6$.

$$x^{10} \cdot x^{11} = x^{21}$$

$$\frac{10^k}{100} = \frac{10^k}{10^2} = 10^{k-2}$$

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- <u>Def</u> A <u>rational number</u> is a real number that can be expressed as a ratio n/m of integers n and m where $m \neq 0$.

<u>ex</u> 1.2 is rational because we can choose $n = 6$ and $m = 5$ so that $1.2 = n/m = 6/5$.

-5 is rational because $-5 = -5/1$.

π is not rational.

$0.33\ldots$ is rational because it equals $1/3$.

- **Def** The absolute value of a number x, written $|x|$, is the distance from x to 0, disregarding the sign of x.

ex $|5| = 5$

$|-5| = 5$

$|-1.2| = 1.2$

- **Def** Given a proposition of the form "if a then b", its converse is the proposition "if b then a."

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- **Def** An integer n is odd iff there is an integer k such that $n = 2k+1$.

ex -11 is odd because $-11 = 2(-6)+1$
10 is not odd because there is no integer k so that $10 = 2k+1$.

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- **Def** A positive integer $n > 1$ is prime if the only positive integers that evenly divide n are one and n itself. A positive integer $n > 1$ is composite if it is not prime.

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- The logarithm base x of y is the number we must raise x to to get y.

ex $\log_2 16 = 4$ because $2^4 = 16$

$\log_{10} 100 = 2$ because $10^2 = 100$

$\log_2 100 = 6.64$

Some log rules:

$\log_b x^k = k \log_b$

$\log_b b = 1$

$\log_b x/y = \log_b x - \log_b y$

$\log_b xy = \log_b x + \log_b y$

$\log_b x = \dfrac{\log_a x}{\log_a b}$ ← change of base rule

- intuitively, log is the opposite of exponentiation (raising something to a power)
- it is very important in CS

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- $f(\cdot)$ notation: just helps us be clear about what f is referring to. For big O, we can write $O(\cdot)$ so we know that something goes in the parentheses.

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- Summation rules:

$$\sum_{i=1}^{n} c = nc$$

constant

$$\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

things that depend on i

$$\sum_{i=1}^{n} n = n^2$$

$$\sum_{i=1}^{n} i = \frac{n(n-1)}{2}$$ we proved this w/ induction!