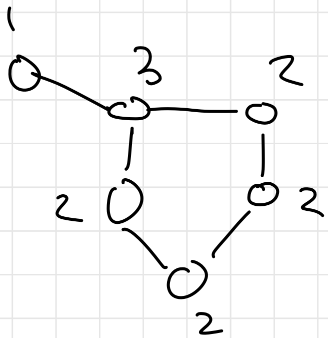
warmup: let's label each node w/ its degree.



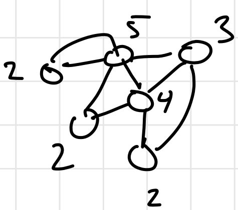
what do we notice about the degrees?

Theorem 11.8 Handshaking Lemma

let $G = (V, E)$ be an undirected graph. Then

$$\sum_{v \in V} \deg(v) = 2|E|.$$

ex

G	$\lvert E \rvert$	$2\lvert E \rvert$	$\sum_{v \in V} \deg(v)$
	0	$2 \cdot 0 = 0$	0
	1	$2 \cdot 1 = 2$	2
	9	$2 \cdot 9 = 18$	$2+2+2+5+4+3 = 18$

Pf let $G = (V, E)$ be an undirected graph. Notice that every edge is incident to exactly two nodes, meaning that it contributes

1 to the degree of exactly 2 nodes. So
$\sum_{v \in V} deg(v) = 2|E|$.

More formally, consider this pseudocode for computing the degrees of all nodes in G:

```
du = 0 for all u∈V
for each edge {u,v} in E:
    du = du + 1
    dv = dv + 1
```

1. Is the algorithm correct?

2. What is $\sum_{u \in V} d_u$ after i iterations of the for loop?

$$\sum_{u \in V} d_u = 2i$$

We run the for loop $|E|$ times, so $\sum_{u \in V} d_u = 2|E|$, and $d_u = deg(u)$ $\forall u \in V$.

Q/
What does the handshaking lemma say about real-life handshakes?

<u>Corollary</u> (fact that follows simply from a previous theorem)

Let n_{odd} denote the number of nodes whose degree is odd. Then n_{odd} is even.

<u>Pf</u> Aiming for a contradiction, suppose that n_{odd} is odd.

$$\sum_{v \in V} \deg(v) = \underbrace{\sum_{\substack{v \in V: \\ \deg(v) \text{ odd}}} \deg(v)}_{\substack{\text{odd bc} \\ \text{odd} \cdot \text{odd is odd}}} + \underbrace{\sum_{\substack{v \in V: \\ \deg(v) \text{ even}}} \deg(v)}_{\substack{\text{even bc} \\ \text{even} \cdot \text{even is even}}}$$

$$= \text{odd} + \text{even}$$
$$= \text{odd}.$$

But this contradicts that $\sum_{v \in V} \deg(v) = 2|E| = \text{even}$.

<u>Q</u> Is the handshaking lemma true for directed graphs? What would the corresponding claim be?