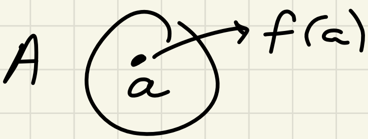
Def let A, B be sets.

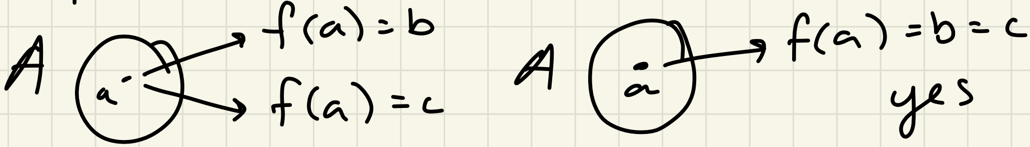
$f: A \to B$ is a <u>function</u> iff it assigns to each $a \in A$ a single value $b \in B$, denoted $f(a)$.

Equivalently, f has the 3 properties:

1) for each $a \in A$, $f(a)$ is defined.



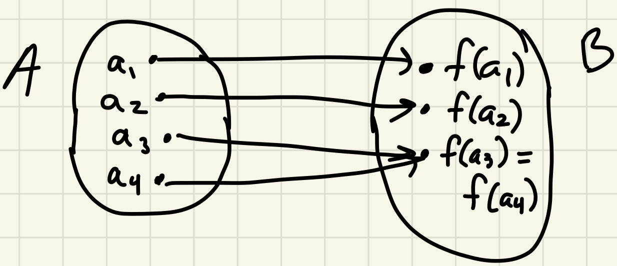
2) for each $a \in A$, $f(a)$ does <u>not</u> produce 2 diff. outputs.



no

and $b \neq c$

3) for each $a \in A$, $f(a) \in B$



A is called the <u>domain</u> of f

B is called the <u>codomain</u> of f

The <u>range</u> of f is $\{f(a) : a \in A\}$

we can represent functions in a table:

$a \in A$	$f(a) \in B$
a_1	$f(a_1)$
a_2	$f(a_2)$
a_3	$f(a_3)$
a_4	$f(a_4)$
$\vdots$	

← some elts of B may have more than one row or zero rows

all elts of A have exactly one row

<u>ex</u> $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.

domain: $\mathbb{R}$
codomain: $\mathbb{R}$
range : $\mathbb{R}^{\geq 0}$

note that $\forall x \in \mathbb{R}$, x^2 is defined (property 1)

$\forall x \in \mathbb{R}$, $f(x) = x^2$, a single value (Property 2)

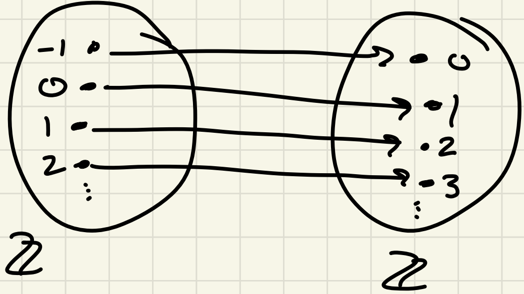
$\forall x \in \mathbb{R}$, $f(x) \in \mathbb{R}$, because $x^2 \in \mathbb{R}$ (Property 3)

<u>ex</u> is $L: \mathbb{R} \to \mathbb{R}$ defined by $L(x) = \log(x)$ a function?

Is it true that $\forall x \in \mathbb{R}$, $L(x)$ is defined? No! $\log(0)$ is undefined.

<u>ex</u> $s: \mathbb{Z} \to \mathbb{Z}$ def. by $s(x) = x+1$ ("successor function")

domain: $\mathbb{Z}$
codomain: $\mathbb{Z}$
range: $\mathbb{Z}$



Prop 1: ✓

Prop 2: for all $a \in \mathbb{Z}$, if $s(a) = b$ and $s(a) = c$, then $b = c$.

Suppose $s(a) = b$ and $s(a) = c$. WTS $b = c$.

$s(a) = a+1 = b$, $s(a) = a+1 = c$ def of s

$b = c$ subs.