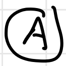
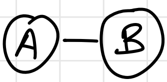
Intro to Graphs

Def An undirected graph $G = (V, E)$ is a non-empty set V of nodes/vertices and a set $E = \{ \{u, v\} : u, v \in V \}$ of edges joining pairs of nodes.

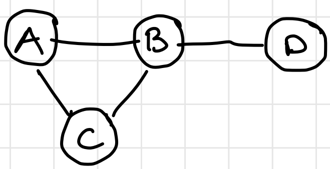
ex.



$V = \{A\}$
$E = \emptyset$



$V = \{A, B\}$
$E = \{ \{A, B\} \} = \{ \{B, A\} \}$



$V = \{A, B, C, D\}$
$E = \{ \{A, B\}, \{B, C\}, \{C, B\}, \{B, D\} \}$

non-ex.

all edges need 2 endpoints

real-word examples:

- facebook friend ↗ nodes = people, edges when they are friends
- blood-related

Q What property would a relation need to be representable as an undirected graph?

Def A directed graph $G = (V, E)$ has set of vertices and edges $E \subseteq V \times V = \{ (u, v) : u, v \in V \}$ so that edges are directed from one vertex to another.

ex A

$A \to B$ $E = \{\langle A, B \rangle\}$

$\neq$

$A \leftarrow B$ $E = \{\langle B, A \rangle\}$

- relations are directed graphs
- functions are directed graphs

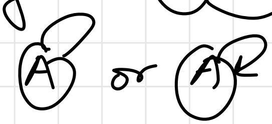
real-world examples:

- twitter followers
- transportation networks

SEA $\to$ BZN

Def A graph is simple if it contains no parallel edges or self-loops.

Parallel edges: 

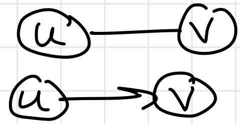
Self-loops : 

Example 11.3: Self-loops and parallel edges.

Suppose that we construct a graph to model each of the following phenomena. In which settings do self-loops or parallel edges make sense?

1. A social network: nodes correspond to people; (undirected) edges represent friendships.
2. The web: nodes correspond to web pages; (directed) edges represent links.
3. The flight network for a commercial airline: nodes correspond to airports; (directed) edges denote flights scheduled by the airline in the next month.
4. The email network at a college: nodes correspond to students; there is a (directed) edge $\langle u, v \rangle$ if u has sent at least one email to v within the last year.

Def



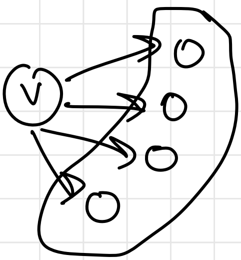
Let $e = \{u, v\}$ or $e = \langle u, v \rangle$

- nodes u, v are <u>adjacent</u> or <u>neighbors</u>
- in a directed graph, v is an <u>out-neighbor</u> of u and u is an in-neighbor of v
- u and v are the <u>endpoints</u> of e
- u and v are <u>incident</u> to e

Let v be a node.

$\text{degree}(v) = \deg(v) = d(v) = \#$ neighbors of v

$= |\{u \in V : \{v, u\} \in E\}|$

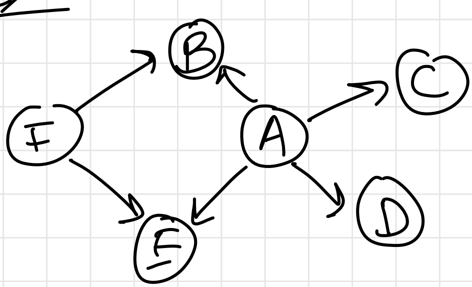


for a directed graph,

$\text{indeg}(v) = \#$ of in-neighbors

$\text{outdeg}(v) = \#$ of out-neighbors

<u>ex</u>



A, B adjacent

D, C not adjacent

A, B are the endpoints of edge $\langle A, B \rangle$

A, E are incident to edge $\langle A, E \rangle$

F is an in-neighbor of B
E is an out-neighbor of A