(4.19)

Claim Suppose $15x + 111y = 55057$ for $x, y \in \mathbb{R}$. Then at least one of x, y is not an integer.

ex	x	y	x int	y int	≥ 1 of x,y not int
	0	$y = \frac{55057}{111}$	T	F	T
	1	$y = \frac{55042}{111}$	T	F	T
	$\frac{55057}{15}$	0	F	T	T

False start at a proof:

$$y = \frac{55057 - 15x}{111} \qquad \text{algebra}$$

$$y = \frac{55057}{111} - \frac{15x}{111} \qquad \text{but now what?}$$

Proof (by contradiction)

Aiming for a contradiction, suppose the claim is false. That is, suppose it is not the case that at least one of x, y is not integer. That is, suppose both x and y are integer.

$55057 = 15x + 111y$ by claim +
$x, y \in \mathbb{Z}$ assumption

$55057 = 3(5x + 37y)$ factoring

$\frac{55057}{3} = 5x + 37y$ algebra

$18352\frac{1}{3} = 5x + 37y$ algebra

$18352\frac{1}{3} \in \mathbb{Z}$ because product, sum of ints is int

But this is a contradiction because $18352\frac{1}{3}$ is not an integer. Therefore, our initial assumption that both $x, y \in \mathbb{Z}$ is false, so at least one is noninteger. □

(4.18 - part)

<u>Claim</u> If n^2 is even, then n is even.

<u>ex</u>	<u>n</u>	n^2	<u>n^2 even?</u>	<u>n even?</u>
	4	16	T	T
	-2	4	T	T
	3	9	F	F

<u>Proof</u> For contradiction, suppose the claim is false. That is, suppose that n^2 is even but n is odd.

$n = 2k+1$ for $k \in \mathbb{Z}$ by def. of odd

$n^2 = (2k+1)^2$ substitution

$n^2 = 4k^2 + 4k + 1$ algebra

$n^2 = 2(2k^2 + 2k) + 1$ factoring

$n^2 = 2c + 1$, $c \in \mathbb{Z}$ — because prod., sum of ints is int

n^2 is odd — def. of odd

This contradicts the fact that n^2 is even, so the assumption that n is odd is false. $\square$

(4.20)

<u>claim</u> $\sqrt{2}$ is not rational.

<u>Proof</u> For the sake of contradiction, assume that $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{n}{d}$, $n, d \in \mathbb{Z}$, $d \neq 0$, n, d in lowest terms (can't have a common divisor) — def. of rational

$2 = \frac{n^2}{d^2}$ — squaring both sides

$2d^2 = n^2$ — algebra

n^2 is even — def. of even

n is even — claim 4.18

$n = 2c$, $c \in \mathbb{Z}$ — def. of even

$n^2 = 4c^2$ — algebra

$n^2 = 4c^2 = 2d^2$ — substitution

$2c^2 = d^2$ — algebra

d^2 is even	def. of even
d is even	claim 4.18

So d and n are both divisible by 2. But this contradicts the fact that n and d have no common divisor. So our initial assumption that $\sqrt{2}$ is rational is false. $\square$