Examples of propositions:

for ints n — $n(n+1)^2$ is even

for ints n — if n^2 even, then n even

for reals x, y — if $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$ then $xy \in \mathbb{Q}$

$\sqrt{2}$ is not rational

In proofs, we've done things like:

n even $\Rightarrow$ $n = 2c$ for $\in \mathbb{Z}$
("implies that")

$nx, ny \in \mathbb{Z} \Rightarrow nx\, ny \in \mathbb{Z}$

$\sqrt{2}$ rational $\Rightarrow \cdots \Rightarrow \cdots \Rightarrow$ false (contradiction)

n int $\Rightarrow$ n is even or odd

We can construct compound propositions out of smaller (atomic) propositions.
can't be broken down any smaller

Sytax vs. Semantics
↳ grammatically correct (for given language)
↳ meaning of a gramatically correct sentence or statement

Let p, q be propositions.

natural lang.	syntax	informal semantics
p and q	$p \wedge q$	T iff both p, q T
p or q	$p \vee q$	T iff ≥1 of p, q T
not p	$\neg p$	T iff p is F
if p then q	$p \Rightarrow q$	T iff when p T, q T
p if and only if q	$p \Leftrightarrow q$	T iff p, q match
p exclusive or q	$p \oplus q$	T iff p, q mismatch

formal semantics

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

ex

2 is even	and	3 is odd	T
2 is even	and	4 is odd	F
1 is even	and	3 is odd	F
3 is even	and	2 is odd	F
2 is even	or	3 is odd	T
2 is even	or	4 is odd	T
1 is even	or	3 is odd	T
3 is even	or	2 is odd	F

not (2 is even) F
not (2 is odd) T

if/then

true iff p "forces" q
it's a promise that whenever p T, q also T
so $p \Rightarrow q$ is F when that promise is broken.
That is, when p is T and q is F.

<u>ex</u> if it rains then the grass is wet.

when is this a lie?

rains	grass wet	
T	T	✓
T	F	✗
F	T	✓
F	F	✓

if p then q can also be written as:

- q whenever p
- q is necessary for p
- p only if q
- p is a sufficient condition for p
- whenever p also q
- p implies q

<u>Q</u> suppose we have propositions p, q, r.
how many rows does the truth table have?

2^n. one for each of $\{T,F\}^n$. (recall set notation)

$\{T,F\} \times \{T,F\} = \{\langle T,T\rangle, \langle T,F\rangle, \langle F,T\rangle, \langle F,F\rangle\}$

$\{T,F\}^3 = \{\langle T,T,T\rangle, \langle T,T,F\rangle, \ldots\}$

Def 2 propositions are <u>logically equivalent</u>, written $\equiv$, iff their truth tables are the same.

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

$p \equiv \neg\neg p$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$\neg p \vee q$	$\neg q \Rightarrow \neg p$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$(p \Rightarrow q) \equiv (\neg p \vee q)$

$(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$

$(p \Rightarrow q) \not\equiv (q \Rightarrow p)$

$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

$\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$

De Morgan's Law

Precedence rules

1. $\neg$
2. $\vee, \wedge, \oplus$
3. $\Rightarrow$
4. $\Leftrightarrow$

Def Prop p is satisfiable iff it is true under at least one truth assignment. That is, at least one row of the truth table evaluates to T.

ex $p \vee \neg p$

$p \vee q$

Def A prop. is a tautology if every row of the truth table is true.

ex $(p \Rightarrow q) \wedge p$