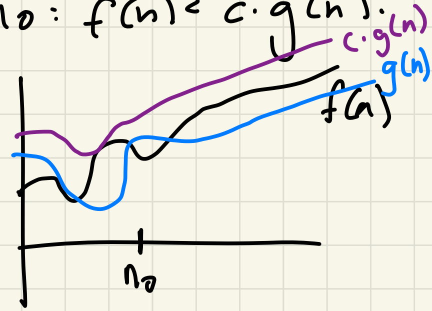
Recall:

For functions $f(n), g(n)$, $f(n) = O(g(n))$
if $\exists\, c > 0, n_0 \geq 0$ s.t. $\forall n \geq n_0 : f(n) < c \cdot g(n)$.



	n_0	c
ex $n = O(n)$	0	3
	0	1
$2n = O(n)$	0	2
	0	3
$n + 8 = O(n)$	4	3
	8	2
$10 = O(n)$	1	10
	3.4	3
$\log_b(n^k) = O(\log n)$	1	$k/\log b$

<u>Lemma 6.2</u> Asymptotic Equivalence of Max + Sum

$f(n) = O(g(n) + h(n)) \iff f(n) = O(\max(g(n), h(n)))$

<u>ex</u> $f(n) = n^2 + n = O(n^2 + n)$

By lemma 6.2, $f(n) = O(\max(n^2, n))$

So $f(n) = O(n^2)$.

This lemma is what allows us to drop lower-order terms!

Pf ($\Rightarrow$)

Suppose $f(n) = O(g(n) + h(n))$. WTS $f(n) = O(\max(g(n), h(n)))$.

$\exists\, c>0,\ n_0 \geq 0 : \forall n \geq n_0 :$ def. of O

$$\begin{aligned} f(n) &\leq c \cdot [g(n) + h(n)] \\ &\leq c \cdot [\max(g(n), h(n)) + h(n)] \\ &\leq c \cdot [\max(g(n), h(n)) + \max(g(n), h(n))] \\ &= 2c \cdot \max(g(n), h(n)) \end{aligned}$$

So $\forall n \geq n_0 : f(n) \leq 2c \cdot \max(g(n), h(n))$.

Which means that $f(n) = O(\max(g(n), h(n)))$ by choosing $n_0' = n_0$ and $c' = 2c$.

($\Leftarrow$) We will do in small groups.

<u>Lemma 6.3</u> Transitivity of $O(\cdot)$

If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

<u>ex</u> $f(n) = 3n$ $\quad$ $f(n) = O(g(n)) \Rightarrow f(n) = O(h(n))$

$g(n) = 2n+2$ $\quad$ $g(n) = O(h(n))$

$h(n) = 4n^2$

<u>Lemma 6.4</u> Addition and multiplication preserve $O(\cdot)$-ness.

If $f(n) = O(h_1(n))$ and $g(n) = O(h_2(n))$,

then $f(n) + g(n) = O(h_1(n) + h_2(n))$ and
$f(n) \cdot g(n) = O(h_1(n) \cdot h_2(n))$.

Note: not true for $-$, $/$

$f(n) = n^3$
$g(n) = n^2$

$f(n) = O(n^3)$
$g(n) = O(n^3)$

$f(n) - g(n) = n^3 - n^2$

is $n^3 - n^2 = O(\overset{\text{zero}}{0})$?

<u>Lemma 6.5</u> Let $p(n) = \sum_{i=0}^{k} a_i n^i$

$= a_k n^k + a_{k-1} n^{k-1} \dots a_1 n + a_0$

be a polynomial. Then $p(n) = O(n^k)$.

Proof of 6.3 in HW; pfs of 6.4, 6.5 exercise.

<u>Common Distinct Functions</u>

name	$f(n)$	$O(\cdot)$
constant	$c \in \mathbb{R}^{\geq 0}$	$O(1)$
log	$\log_b n$, $b \in \mathbb{R}^{>1}$	$O(\log n)$
linear	$c \cdot n$, $c \in \mathbb{R}^{\geq 0}$	$O(n)$
n-log-n	$n \log_b n$	$O(n \log n)$

quadratic	deg-2 polynomial $c_2 n^2 + c_1 n + c_0$	$O(n^2)$
cubic	deg-3 poly	$O(n^3)$
$\vdots$		
deg-k poly		$O(n^k)$
exponential	2^n	$O(2^n)$
	3^n	$O(3^n)$
	$\vdots$	$\vdots$

Takeaway: O is an upper <u>bound</u> on functions.

Note: we won't cover other types of asymptotic analysis (Ω, Θ, ω, o), but the book does in 6.2.2.