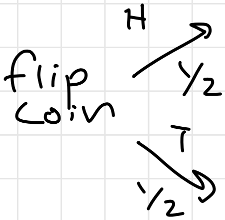
Tree Diagrams in Probability

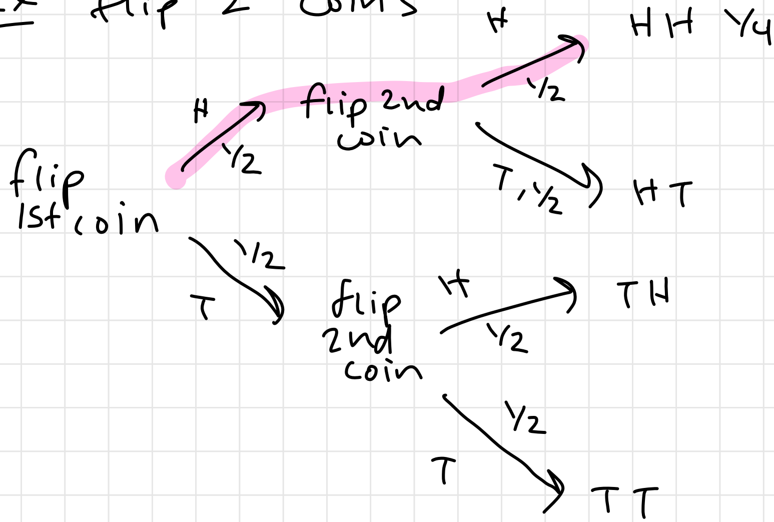
- internal nodes = random choice
  - label w/ probability



sum of probs. = 1

- leaves are outcomes

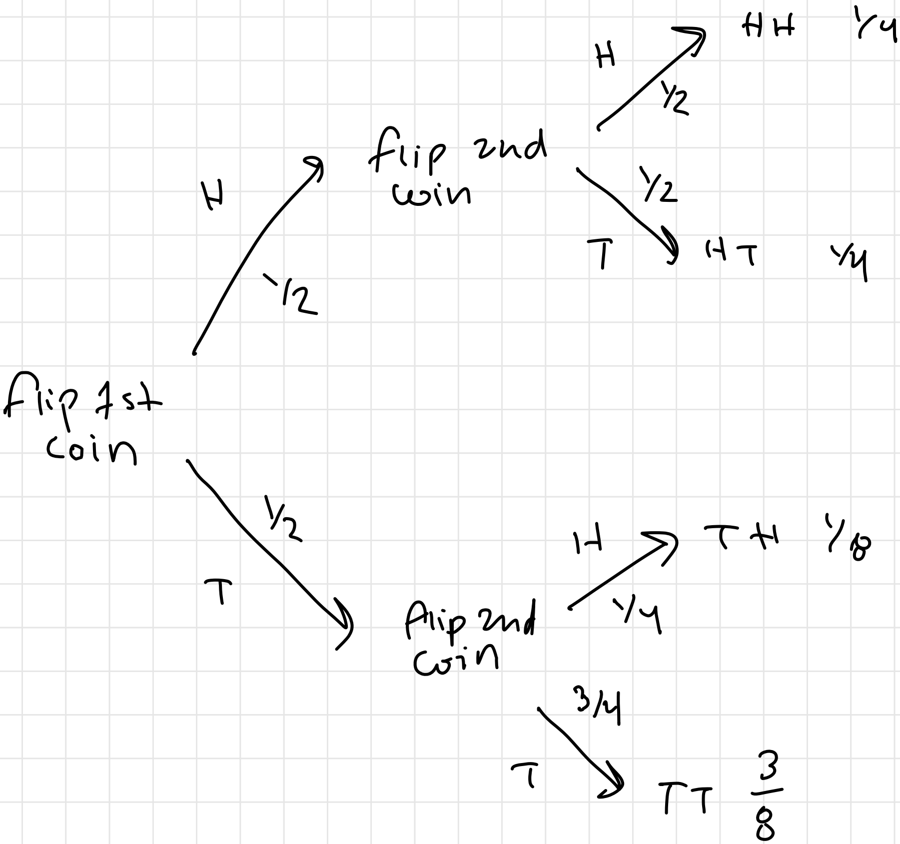
ex flip 2 coins



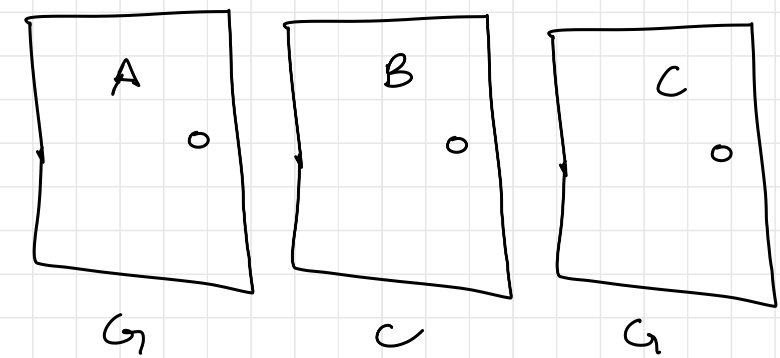
prob of outcome is product of labels back to root

ex flip 1 fair coin. If H, flip 2nd fair coin. If T, flip coin w/ 0.75 prob of T.

- $\Pr[\langle T, T \rangle] = 3/8$
- $\Pr[\text{at least one H}] = 1/4 + 1/4 + 1/8$ or $1 - \frac{3}{8}$



<u>ex</u> Monty hall problem

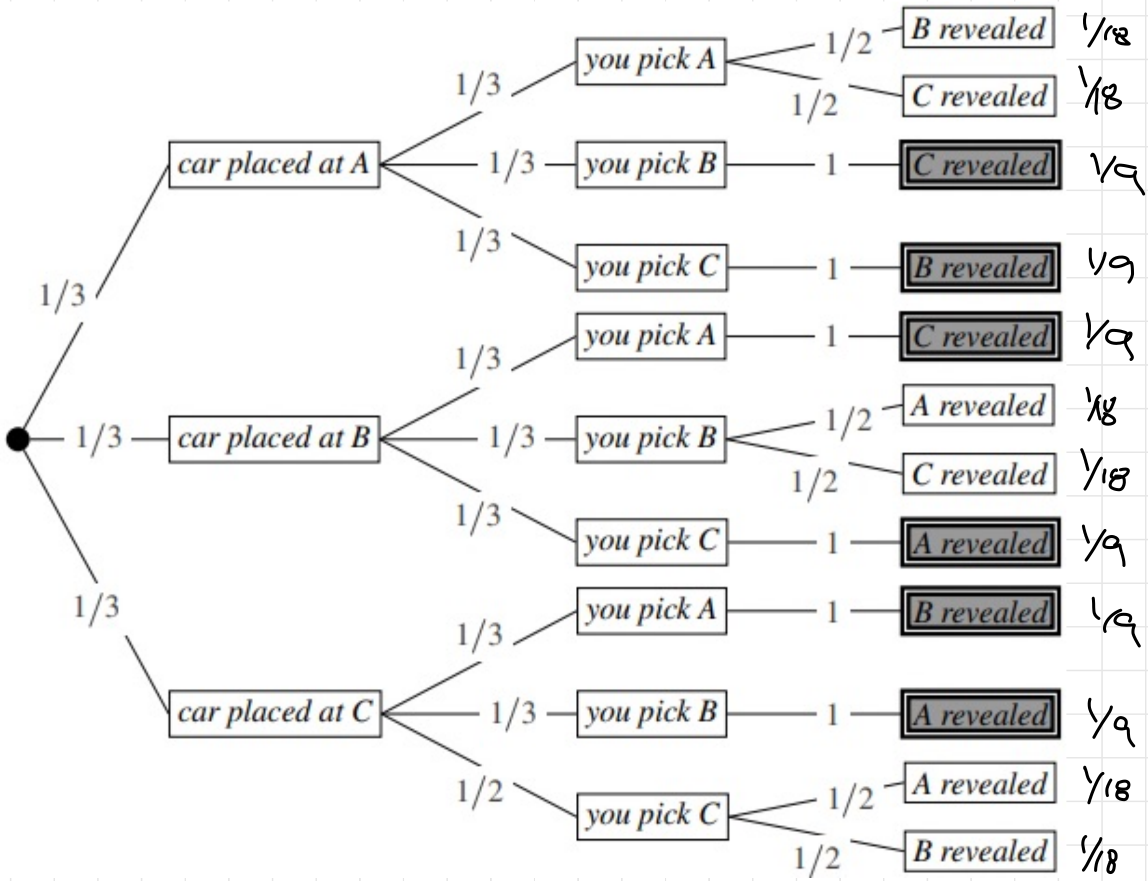


3 doors.
2 have goats,
1 has car

You pick 1 door.

A goat door is revealed.

Should you switch doors?



let S = all outcomes.

car at A, you pick A, B revealed $\in S$

car at A, you pick B, C revealed $\in S$

let $A \subseteq S$ be all outcomes where you win by switching.

what is $\Pr[A]$? $6/9 = 2/3$

what is $\Pr[\bar{A}]$? $3/9 = 1/3$

Def a <u>permutation</u> of a set S is a $|S|$ sequence of elements of S with no repetitions.

ex $S = \{1, 2, 3, 4\}$

$\langle 1, 2, 3, 4 \rangle$ ✓
$\langle 2, 3, 4, 1 \rangle$ ✓
$\langle 2, 2, 4, 1 \rangle$ ✗
$\langle 3, 4, 1 \rangle$ ✗

Thm 9.8 Let S be a set and $|S| = n$. The number of permutations is $n!$

Proof #1. by product rule.

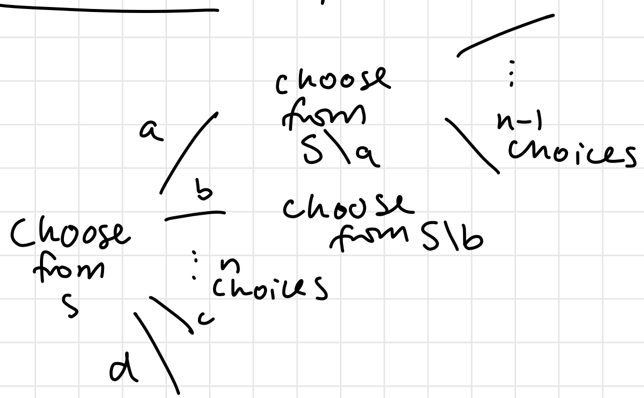
Let S_1 be $S \setminus$ first choice, S_2 be $S_1 \setminus$ second choice, etc.

$$|S \times S_1 \times S_2 \times \cdots \times S_{n-1}| = |S| \cdot |S_1| \cdot |S_2| \cdots |S_{n-1}|$$ by prod. rule

$$= n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot (1)$$

$$= n!$$

Proof # 2: w/ a tree diagram.



$n!$ leaves

Def Let n, k be nonnegative integers w/ $k \leq n$.
Then
$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{"n choose k"}$$

Choosing k items from n

let $S = \{1, 2, 3, 4, 5\}$, $k = 3$

how to select k items from S?

	repetition allowed	no repetition allowed
order matters	n^k	$\frac{n!}{(n-k)!}$
order doesn't matter	$\binom{n+k-1}{k}$	$\binom{n}{k}$