Name _____
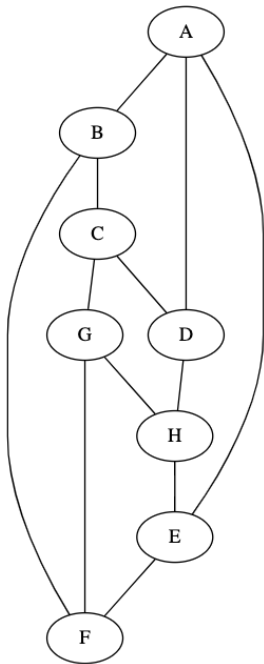
Problem 1 (20 points)

Consider the relation $\geq$ on integers. That is, for all $x, y \in \mathbb{Z}$, $x \geq y$ if the value of x is greater than or equal to the value of y.
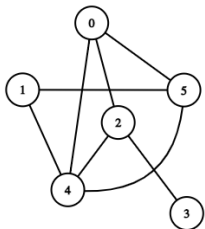
- Is it reflexive?

- Is it irreflexive?

- Is it symmetric?

- Is it anti-symmetric?

- Is it transitive?

- Is it an equivalence relation?

- Is it a partial order?

- Is it a total order?

Problem 2 (20 points)

(a) Is the following graph bipartite? (If yes, label the nodes as belonging to L or R.)

(b) Is the following graph planar? (If yes, re-draw it to demonstrate.)

(c) Draw K_4, the complete graph on four nodes.

(d) Draw a directed graph that contains a cycle.

Problem 3 (20 points)

(a) Let $A = \langle 1, 2, 3, 4, 5 \rangle$ be a 5-element array. How many permutations of A are there?

(b) Suppose we draw a 5-card hand from a standard deck with 52 cards. Define the random variable X to mean the number of aces in the hand. What is $X(\{A\heartsuit, 2\clubsuit, J\clubsuit, Q\diamondsuit, 7\heartsuit\})$?

(c) Now, define X_i to be a random variable that is 1 if the ith card in the list is an ace and 0 otherwise.

What is $X_1(\{A\heartsuit, 2\clubsuit, J\clubsuit, Q\diamondsuit, 7\heartsuit\}$?

What is $X_5(\{A\heartsuit, 2\clubsuit, J\clubsuit, Q\diamondsuit, 7\heartsuit\}$?

(d) For $i \in \{1, 2, 3, 4, 5\}$, what is $E[X_i]$?

(e) By linearity of expectation, $E[X_1 + X_2 + X_3 + X_4 + X_5] = E[X_1] + E[X_2] + E[X_3] + E[X_4] + E[X_5]$. Use this fact to compute the expected number of aces in the hand.

Problem 4 (40 points)

(a) Suppose you draw 2 cards from a standard 52-card deck. How many distinct hands are possible?

(b) How many different 8 bit strings exist? (For example, 00101110 is an 8 bit string.)

(c) Let $f : A \to B$ be a function. Consider the graph $G = (V, E)$ where $V = A \cup B$ and an edge exists from node a to b if $b = f(a)$. Suppose f is one-to-one and onto. Is the proposition $\exists v \in B : indeg(v) = 0$ true or false?

(d) In the following graph, what is the distance from a to f?
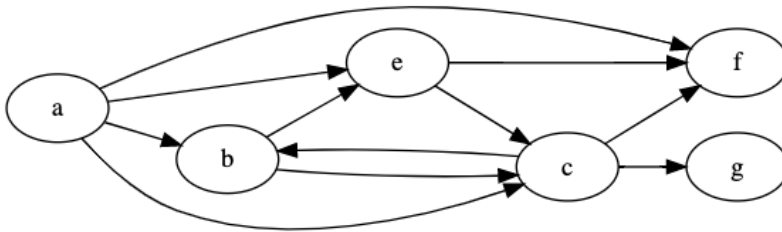
(e) Is the following a valid graph?

(f) Suppose you want to represent the blood relationships between people as a graph. Would this be best represented using a directed graph or an undirected graph?

(g) Give a relation that is both symmetric and anti-symmetric.

(h) What are the properties of a *strict* partial order?

(i) Let $A = \{1, 2, 3, 4\}$ and let R be a relation on A such that for any $a, b \in A$, aRb if a and b have the same parity (i.e., are both even or both odd). R is an equivalence relation. What are its equivalence classes?

(j) Let S be any set and let R be an equivalence relation on S. Suppose that $a \in S$ is not in the equivalence class of $b \in S$; that is, $a \notin [b]$. Is $\langle a, b \rangle \in R$, $\langle a, b \rangle \notin R$, or do we not know?