CSCI 332, Fall 2024

Homework 1

Due before class on Tuesday, September 3, 2024–that is, due at 9:30am Mountain Time

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Submission Requirements

- Type or clearly hand-write your solutions into a PDF format so that they are legible and professional. Submit your PDF on Gradescope.
- Do not submit your first draft. Type or clearly re-write your solutions for your final submission.
- Use Gradescope to assign problems to the correct page(s) in your solution. If you do not do this correctly, we will ask you to resubmit.
- You may work with a group of up to three students and submit ***one single document*** for the group. Just be sure to list all group members at the top of the document. When submitting a group assignment to Gradescope, only one student needs to upload the document; just be sure to select your groupmates when you do so.

Academic Integrity

Remember, you may access ***any*** resource in preparing your solution to the homework. However, you ***must***

- write your solutions in your own words, and
- credit every resource you use (for example: "Bob Smith helped me on this problem. He took this course at UM in Fall 2020"; "I found a solution to a problem similar to this one in the lecture notes for a different course, found at this link: www.profzeno.com/agreatclass/lecture10"; "I asked ChatGPT how to solve part (c)"; "I put my solution for part (c) into ChatGPT to check that it was correct and it caught a missing case.") If you use the provided LaTeX template, you can use the `sources` environment for this. Ask if you need help!

Grading

Remember, submitted homeworks are graded for completeness, not correctness. Correctness is evaluated using homework quizzes.

Each submitted problem will be graded out of six points according to the following rubric:

- Does the solution address the correct problem?
- Does the solution make a reasonable attempt at solving the problem, even if not fully correct?
- Is the presentation neat?
- Is the explanation clear?

- Does the solution list collaborators or sources, or state that the student did not use any collaborators or outside resources?
- Is the solution written in the student's own voice (not copied directly from an outside resource)?

1. (This is problem 1 from Chapter 1 of the textbook)

   Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   True or false? In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) *such that* m *is ranked first on the preference list of* w *and* w *is ranked first on the preference list of* m*.*

2. (This is problem 2 from Chapter 1 of the textbook)

   Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   True or false? Consider an instance of the Stable Matching Problem in which there exists a man m *and a woman* w *such that* m *is ranked first on the preference list of* w *and* w *is ranked first on the preference list of* m*. Then in every stable matching* S *for this instance, the pair* (m, w) *belongs to* S*.*

3. (This is problem 3 from Chapter 1 of the textbook)

   There are many other settings in which we can ask questions related to some type of "stability" principle. Here's one, involving competition between two enterprises.

   Suppose we have two television networks, whom well call $\mathscr{A}$ and $\mathscr{B}$. There are n prime-time programming slots, and each network has n TV shows. Each network wants to devise a *schedule*—an assignment of each show to a distinct slot—so as to attract as much market share as possible.

   Here is the way we determine how well the two networks perform relative to each other, given their schedules. Each show has a fixed *rating*, which is based on the number of people who watched it last year; we'll assume that no two shows have exactly the same rating. A network *wins* a given time slot if the show that it schedules for the time slot has a larger rating than the show the other network schedules for that time slot. The goal of each network is to win as many time slots as possible.

   Suppose in the opening week of the fall season, Network $\mathscr{A}$ reveals a schedule S and Network $\mathscr{B}$ reveals a schedule T. On the basis of this pair of schedules, each network wins certain time slots, according to the rule above. We'll say that the pair of schedules (S, T) is *stable* if neither network can unilaterally change its own schedule and win more time slots. That is, there is no schedule S' such that Network $\mathscr{A}$ wins more slots with the pair (S', T) than it did with the pair (S, T); and symmetrically, there is no schedule T' such that network $\mathscr{B}$ wins more slots with the pair (S, T') than it did with the pair (S, T).

   The analogue of Gale and Shapley's question for this kind of stability is the following: For every set of TV shows and ratings, is there always a stable pair of schedules? Resolve this question by doing one of the following two things:

   (a) give an algorithm that, for any set of TV shows and associated ratings, produces a stable pair of schedules; or

   (b) give an example set of TV shows and associated ratings for which there is no stable pair of schedules.