A criteria that will work

Any ideas?

EARLIEST-FINISH-TIME-FIRST $(n, s_1, s_2, \ldots, s_n, f_1, f_2, \ldots, f_n)$

SORT jobs by finish times and renumber so that $f_1 \leq f_2 \leq \ldots \leq f_n$.

$S \leftarrow \varnothing$.

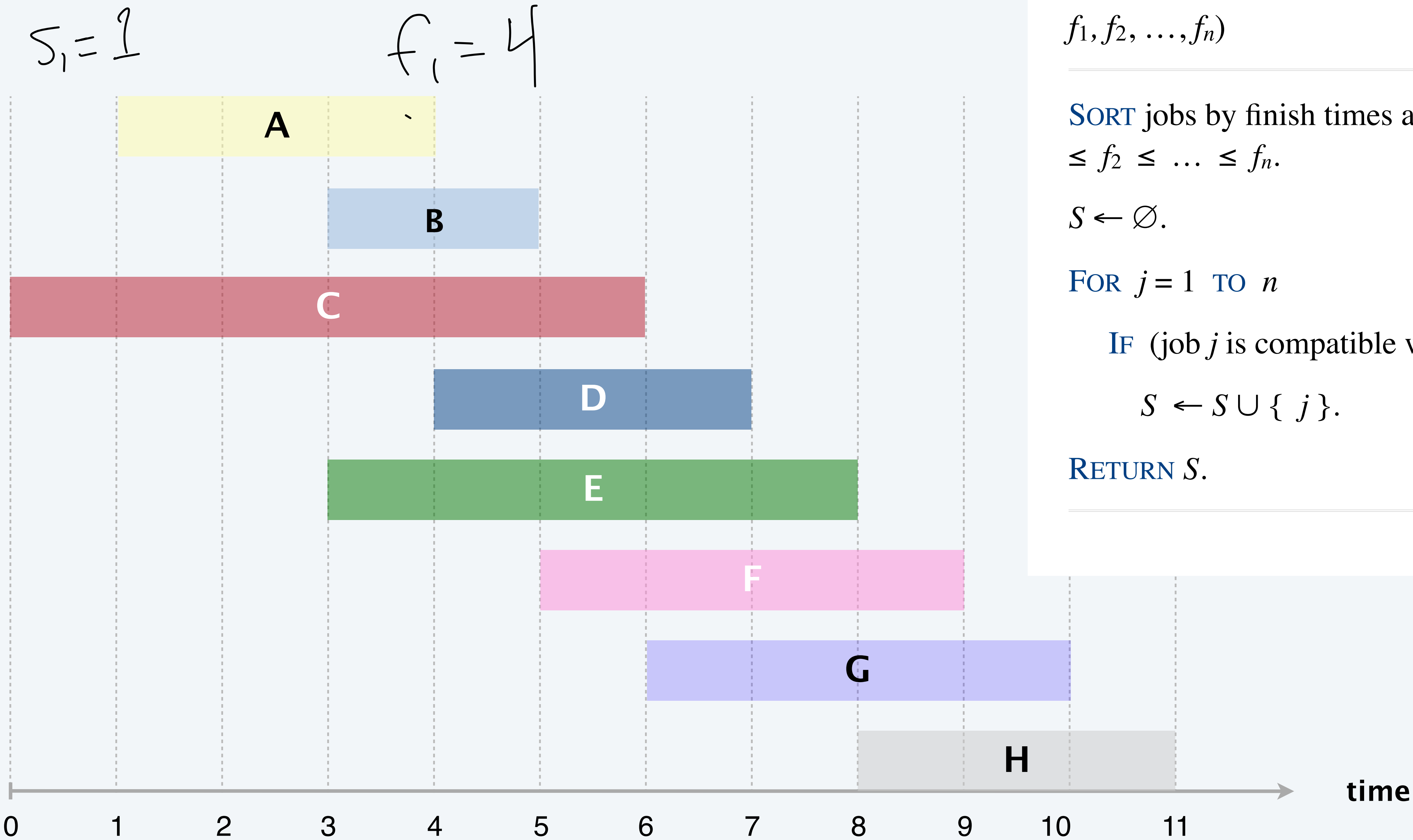
FOR $j = 1$ TO n

  IF (job j is compatible with S)

    $S \leftarrow S \cup \{ j \}$.

RETURN S.

Earliest-finish-time-first algorithm demo



EARLIEST-FINISH-TIME-FIRST $(n, s_1, s_2, \ldots, s_n, f_1, f_2, \ldots, f_n)$

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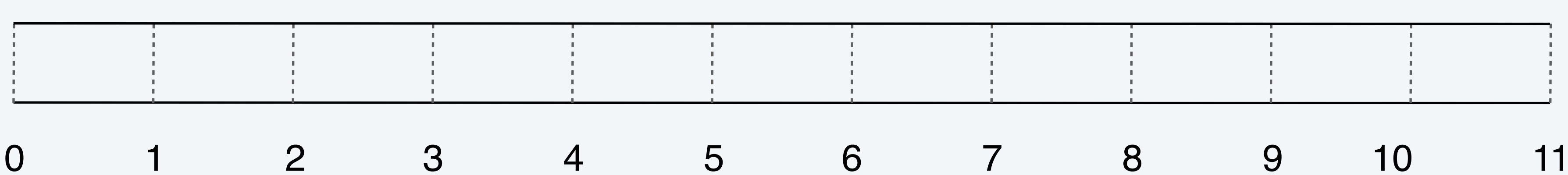
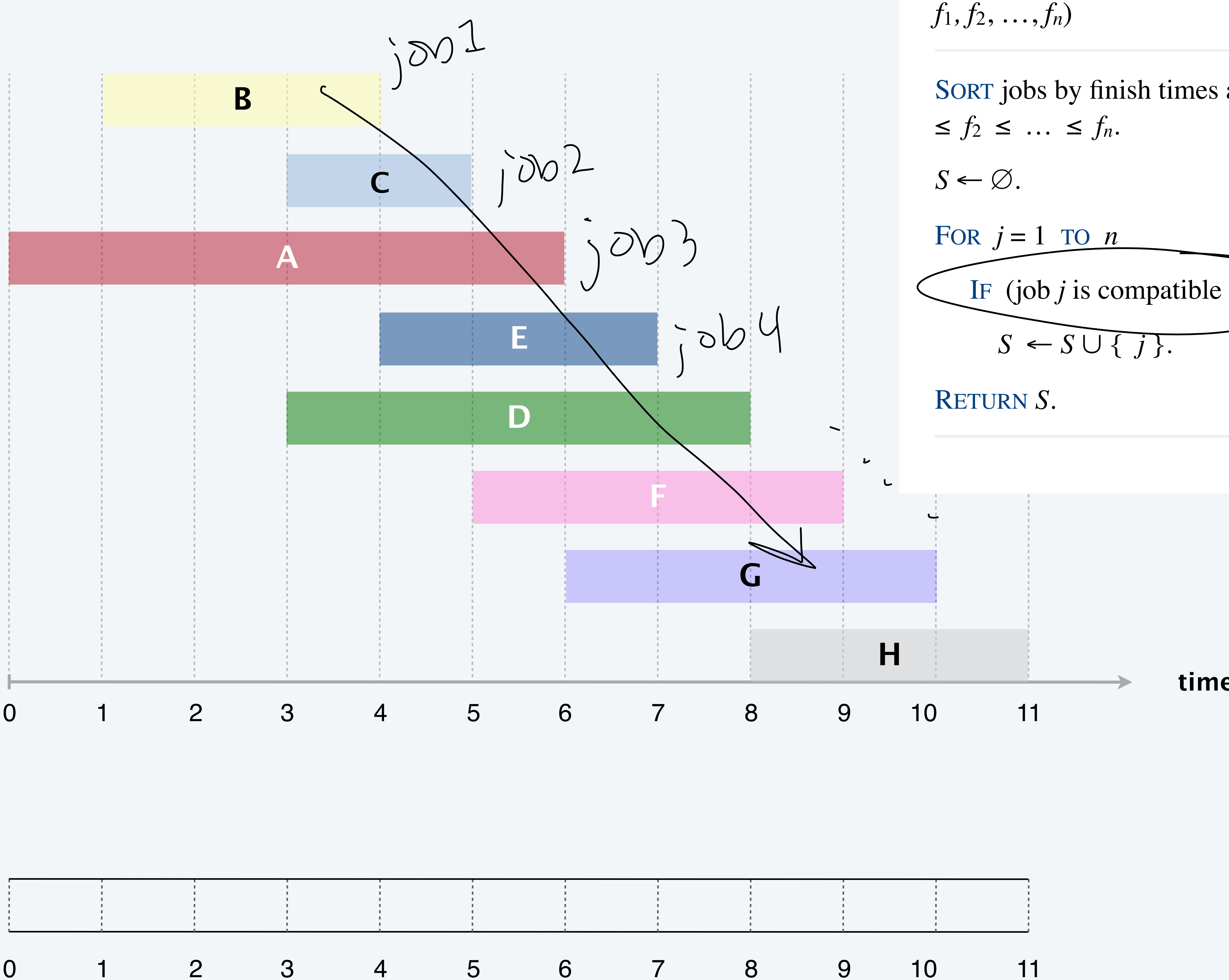


Earliest-finish-time-first algorithm demo



EARLIEST-FINISH-TIME-FIRST $(n, s_1, s_2, \ldots, s_n, f_1, f_2, \ldots, f_n)$

SORT jobs by finish times and renumber so that $f_1 \le f_2 \le \ldots \le f_n$.

$S \leftarrow \varnothing$.

FOR $j = 1$ TO n

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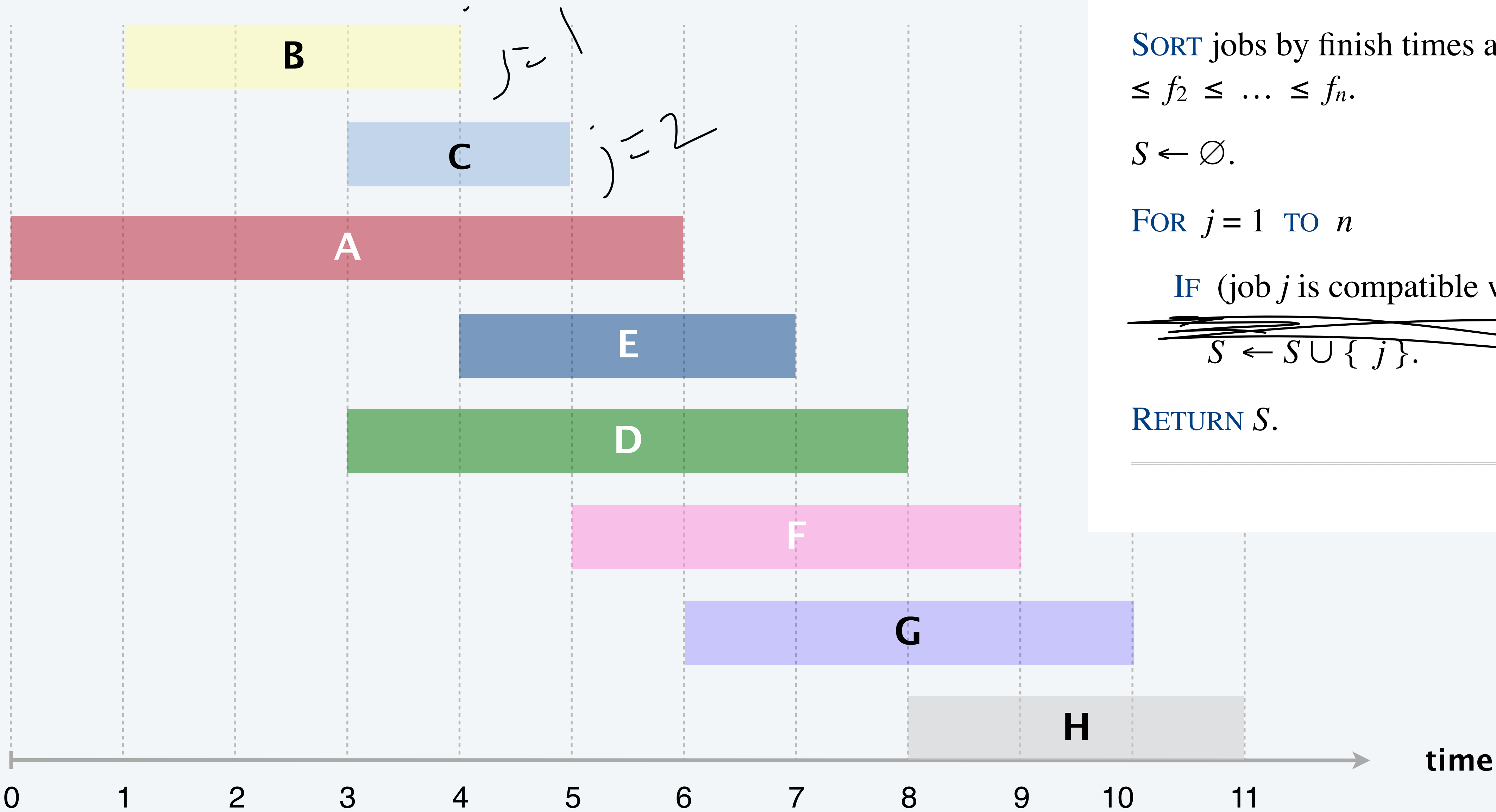
Earliest-finish-time-first algorithm demo



EARLIEST-FINISH-TIME-FIRST $(n, s_1, s_2, \ldots, s_n, f_1, f_2, \ldots, f_n)$

SORT jobs by finish times and renumber so that $f_1 \leq f_2 \leq \ldots \leq f_n$.

$S \leftarrow \varnothing$.

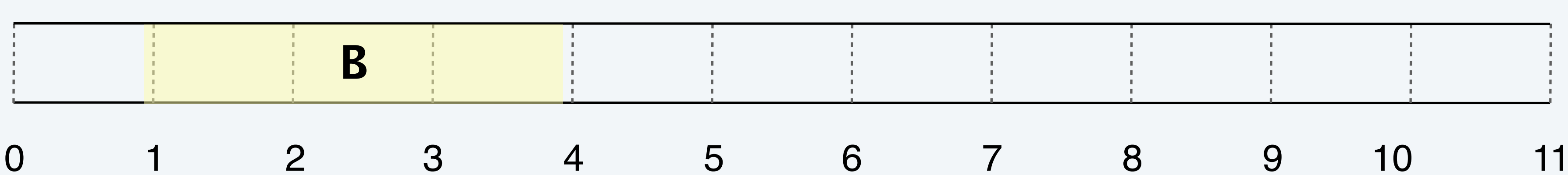
FOR $j = 1$ TO n

  IF (job j is compatible with S)

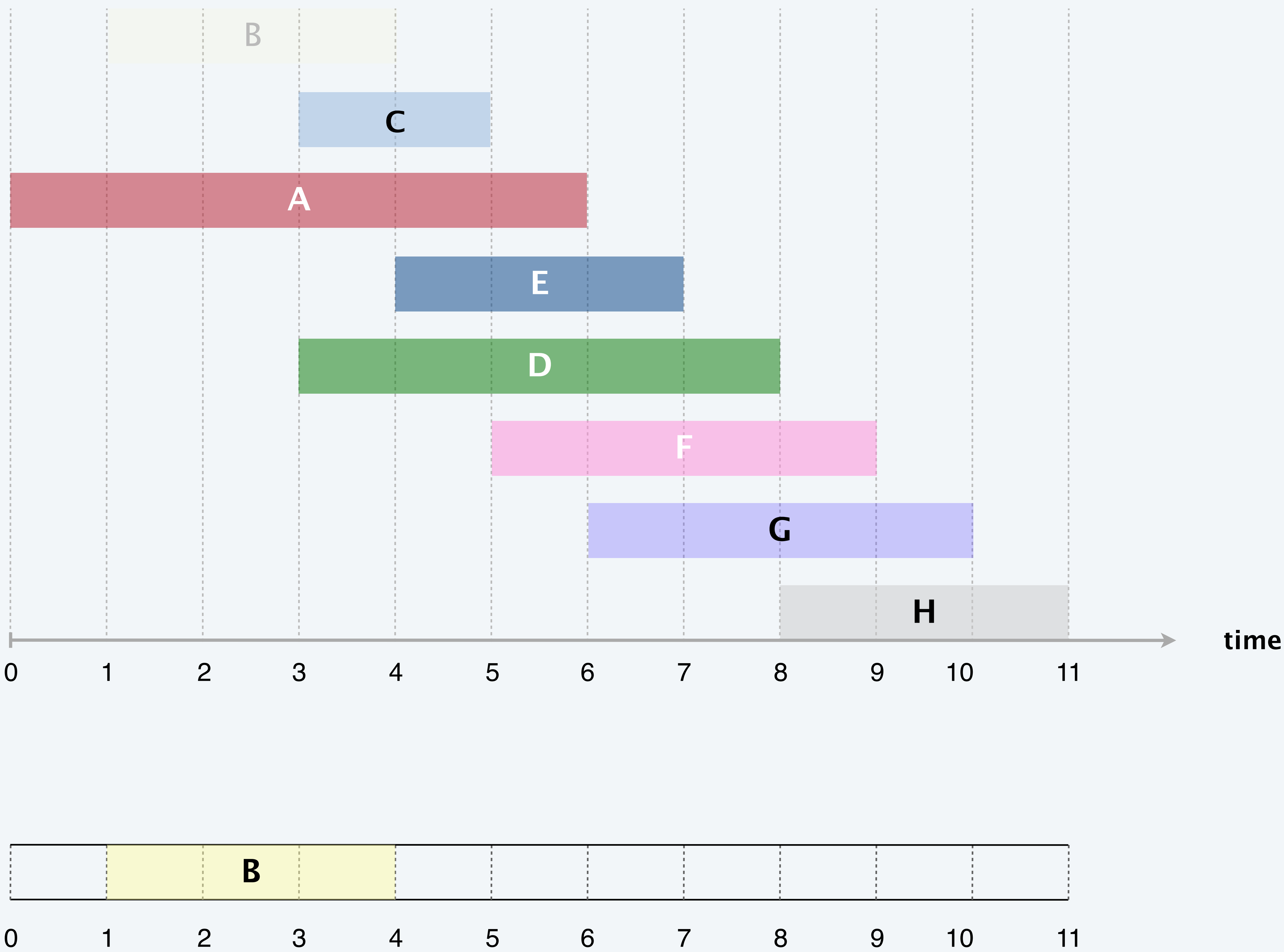
    $S \leftarrow S \cup \{ j \}$.

RETURN S.

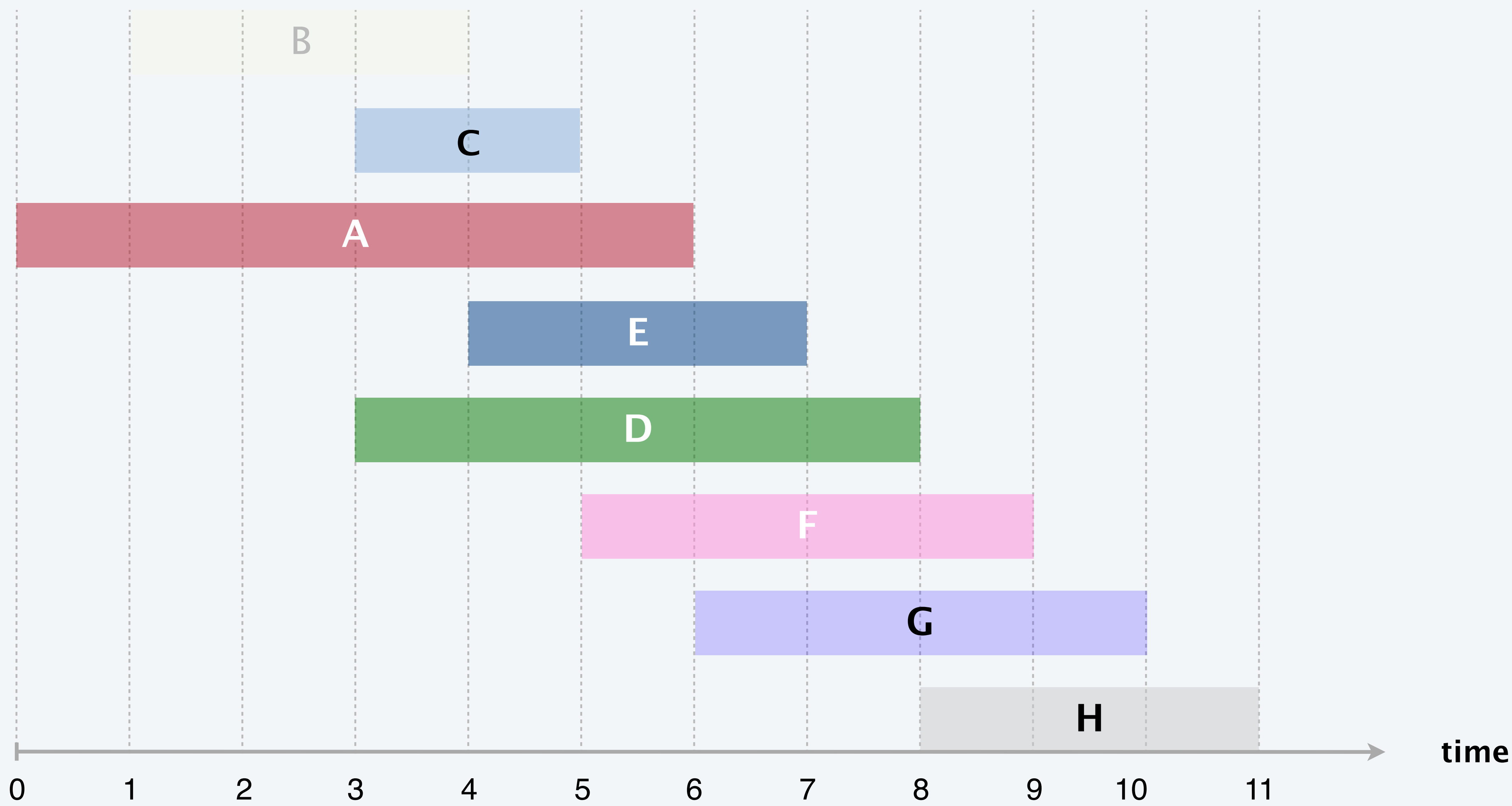
job B is compatible (add to schedule)



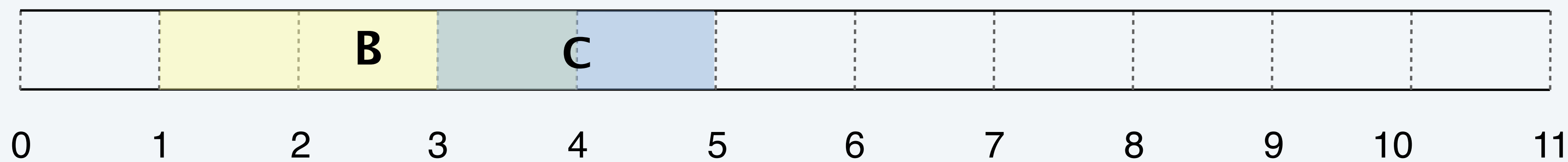
Earliest-finish-time-first algorithm demo



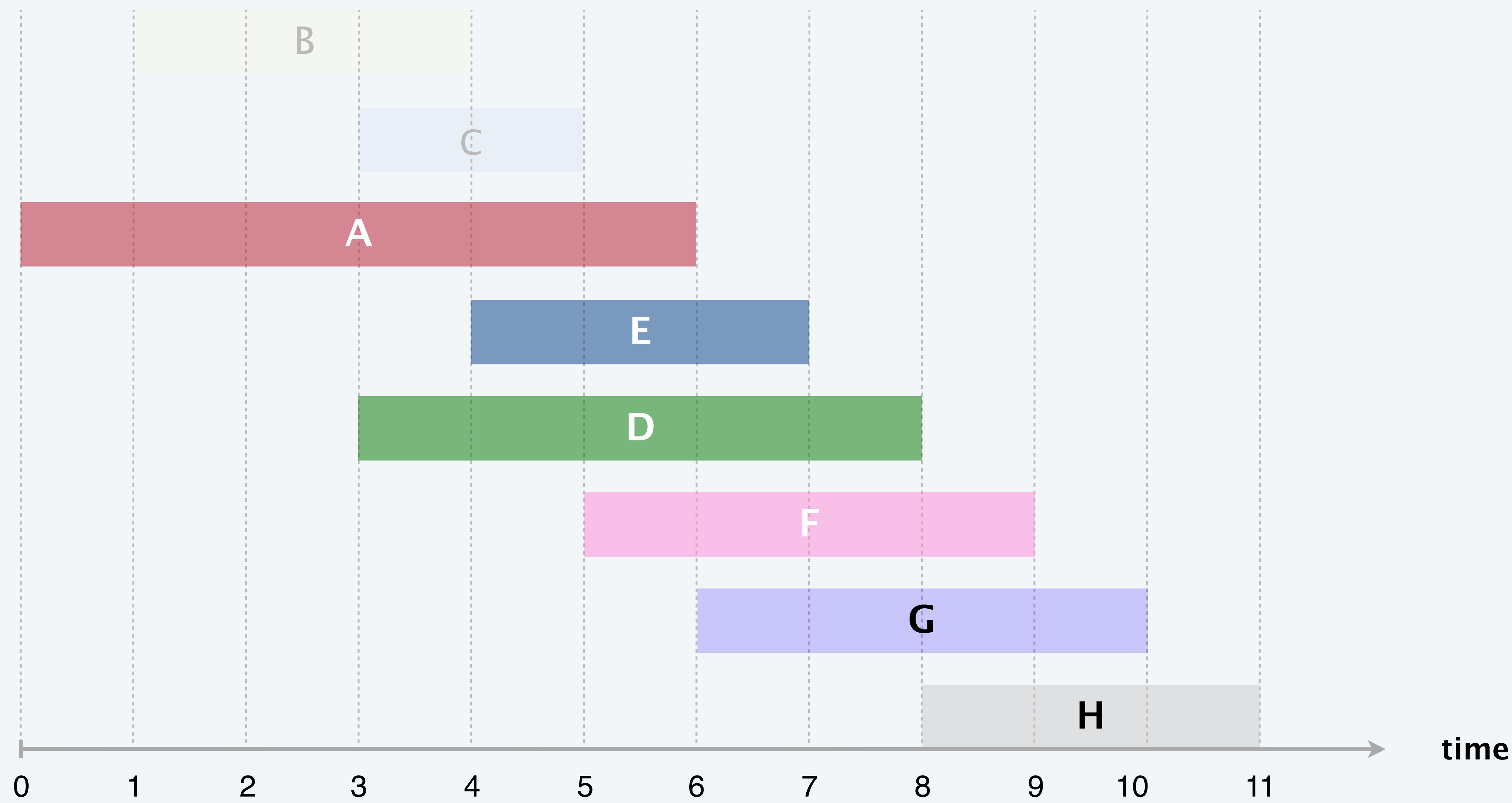
Earliest-finish-time-first algorithm demo

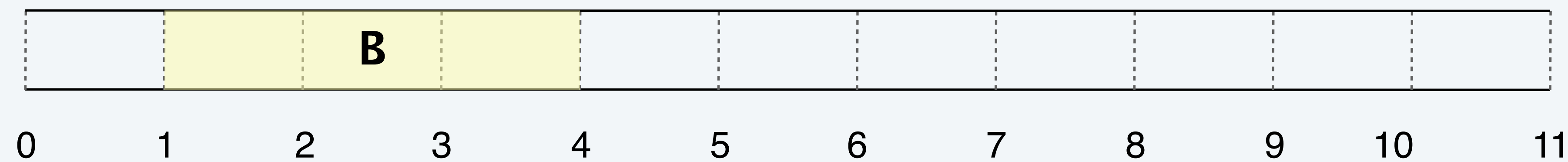


job C is incompatible (do not add to schedule)

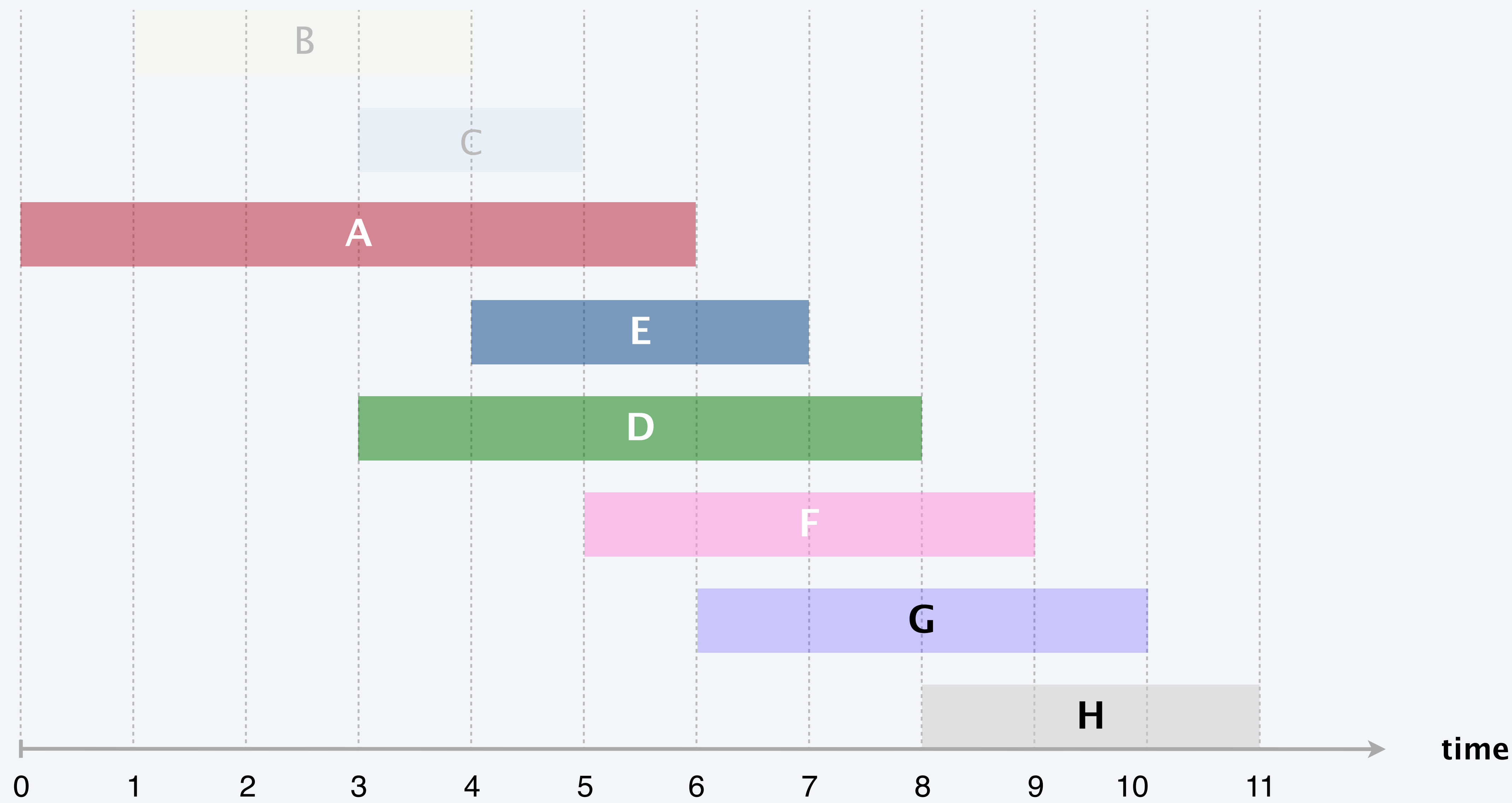


Earliest-finish-time-first algorithm demo

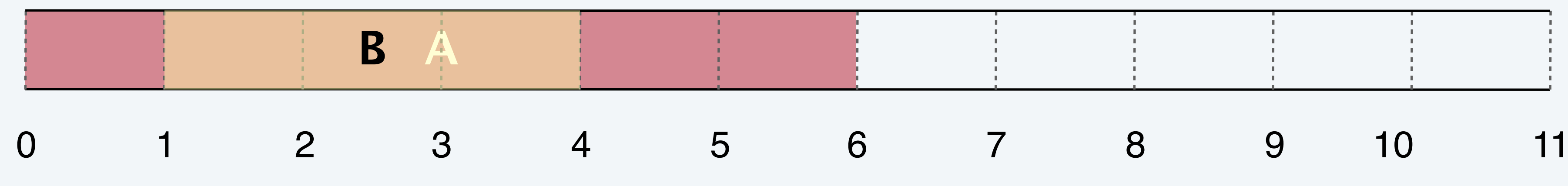




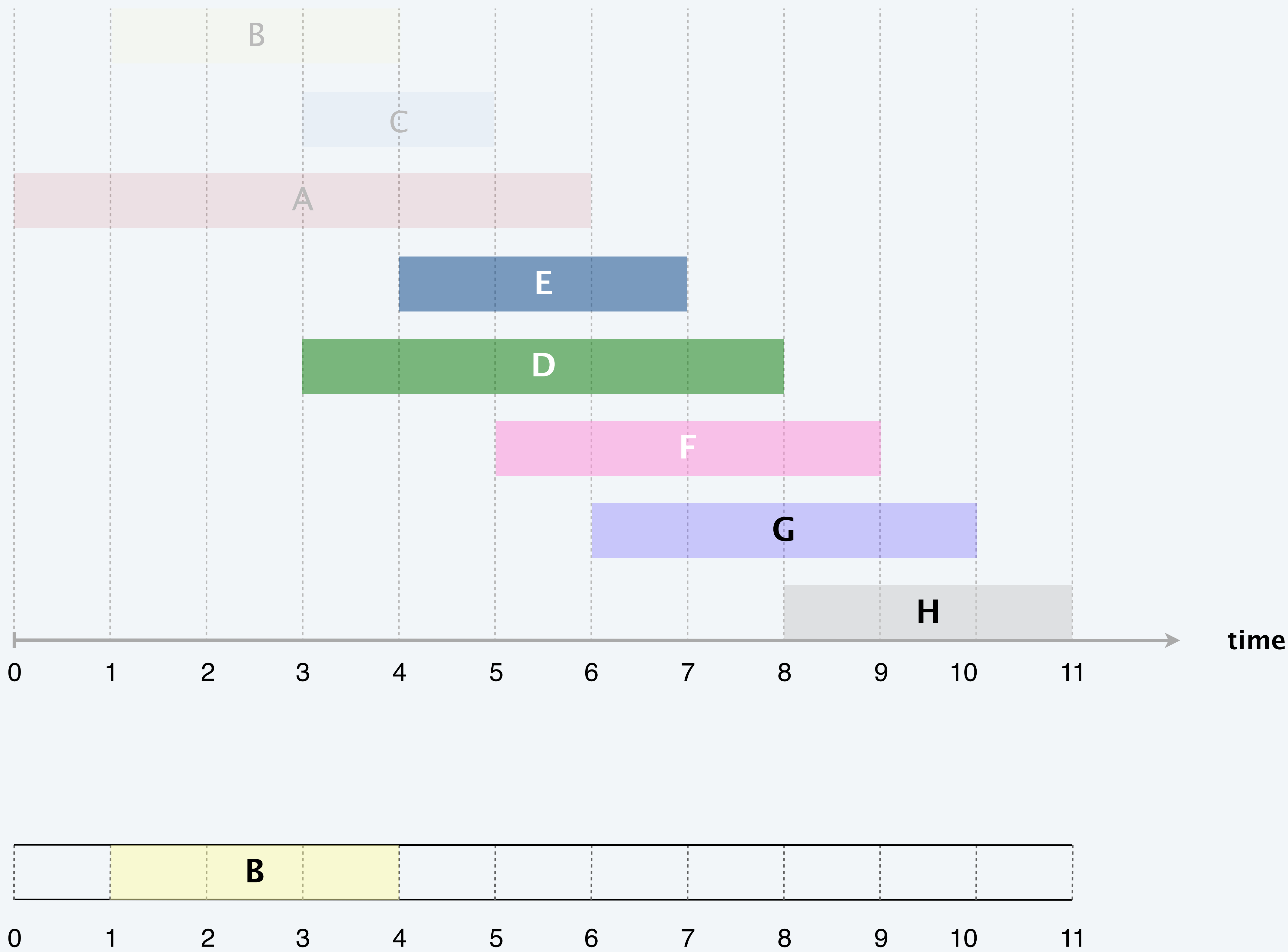
Earliest-finish-time-first algorithm demo



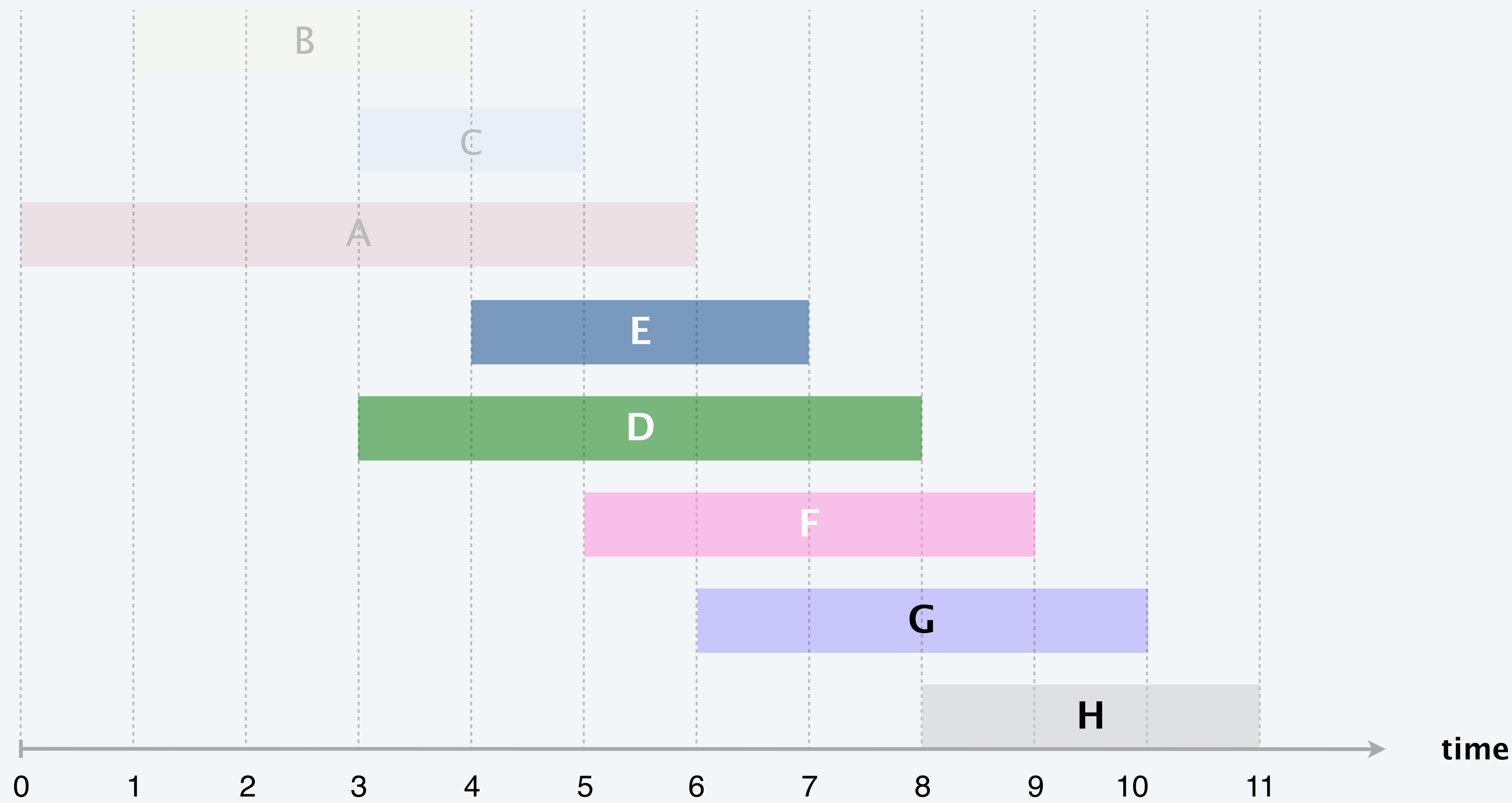
job A is incompatible (do not add to schedule)



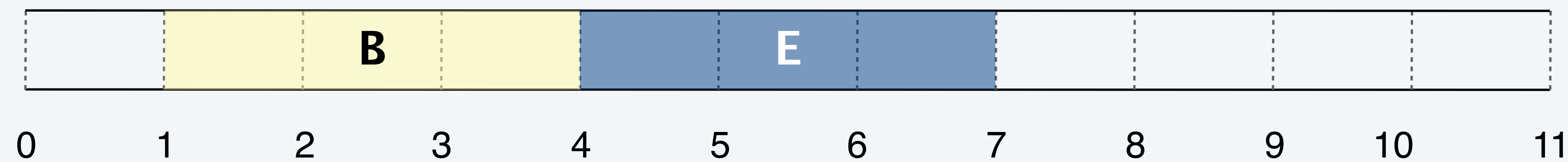
Earliest-finish-time-first algorithm demo



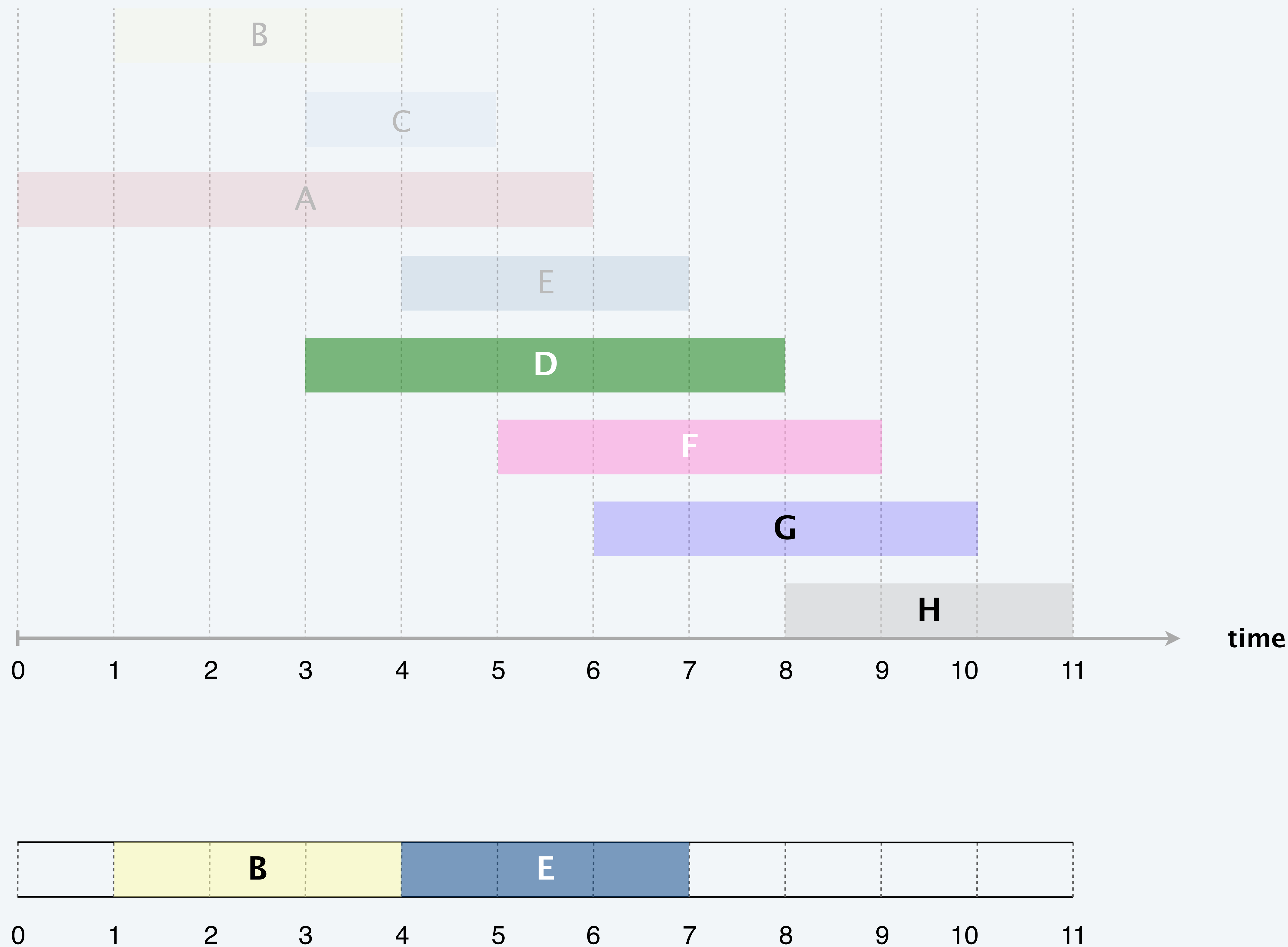
Earliest-finish-time-first algorithm demo



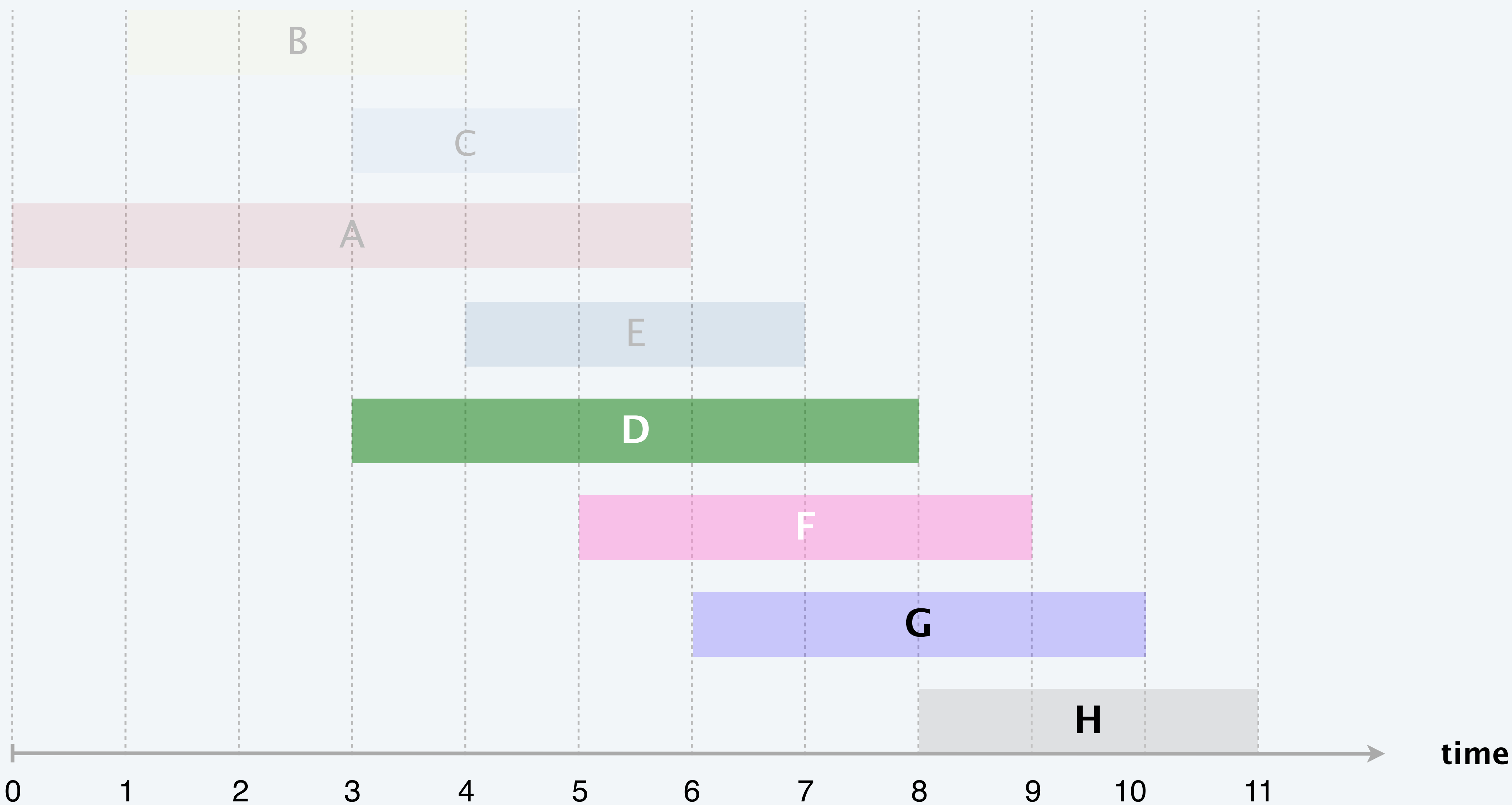
job E is compatible (add to schedule)



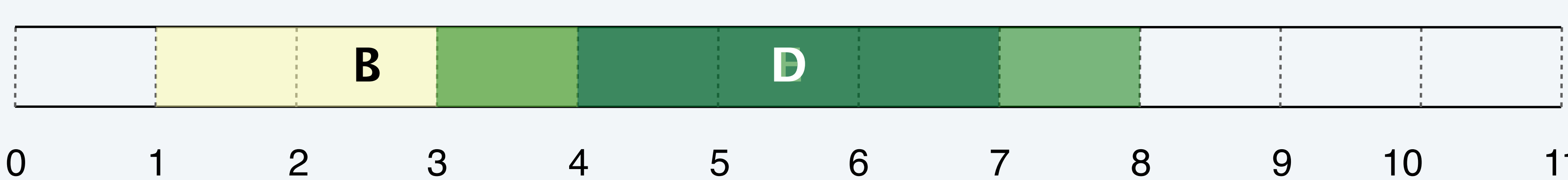
Earliest-finish-time-first algorithm demo



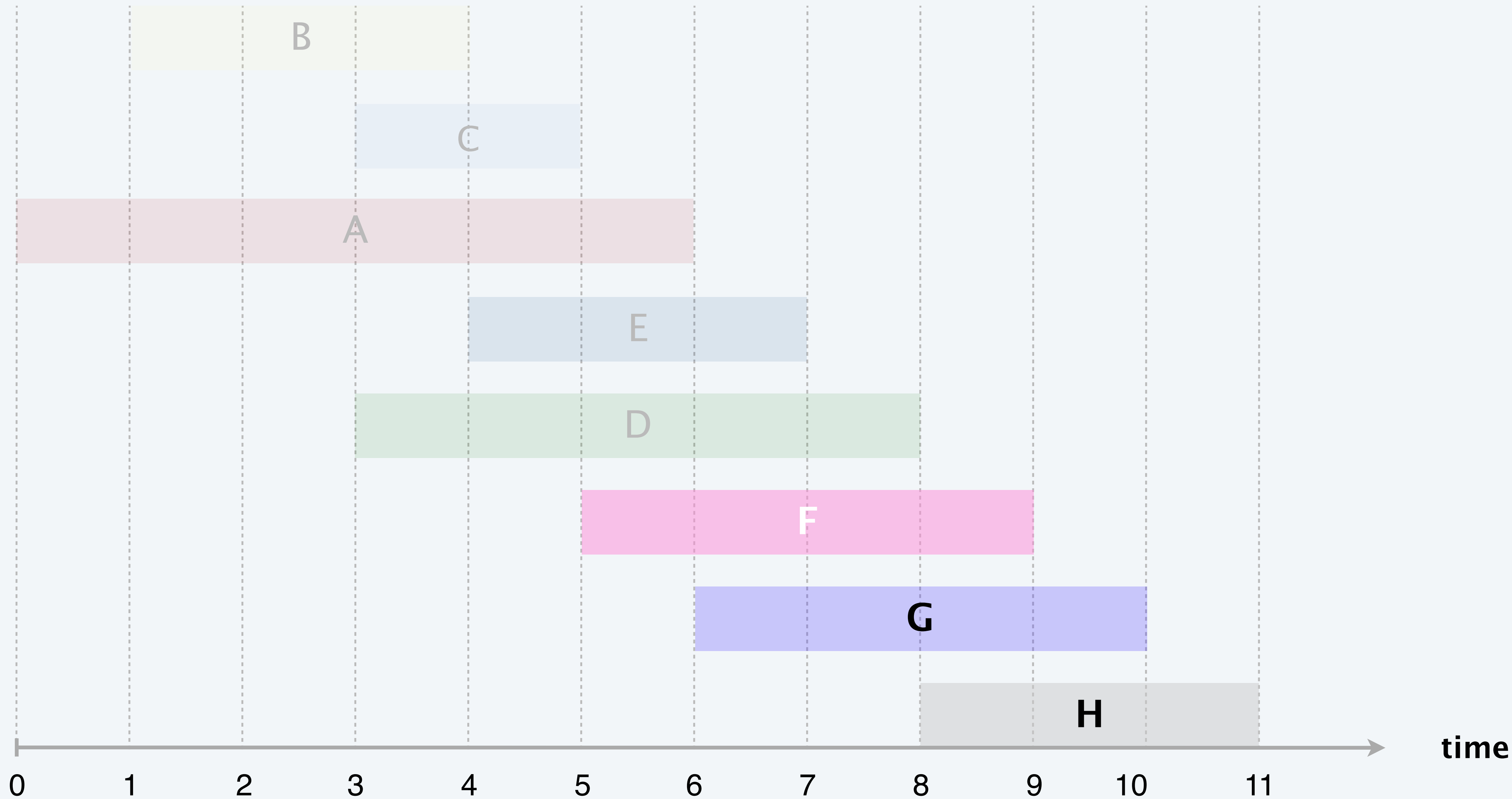
Earliest-finish-time-first algorithm demo

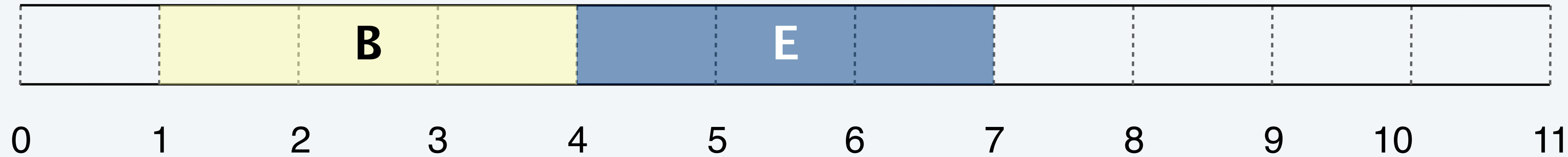


job D is incompatible (do not add to schedule)

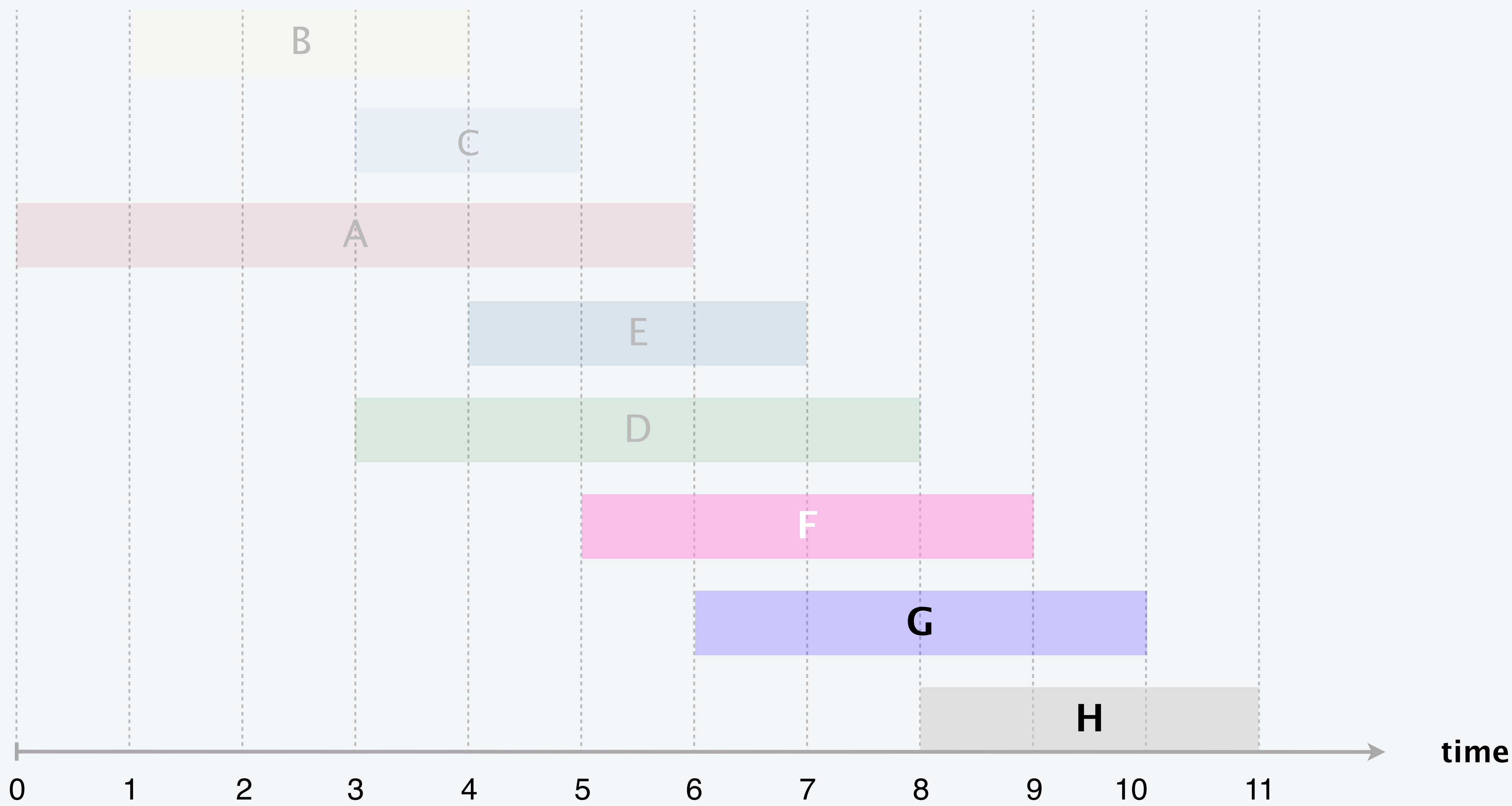


Earliest-finish-time-first algorithm demo

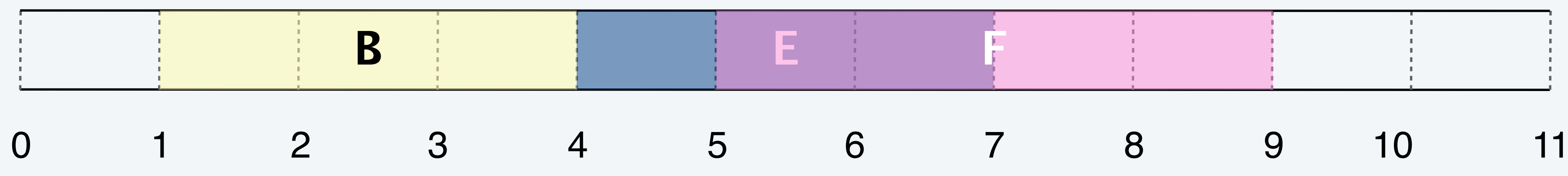




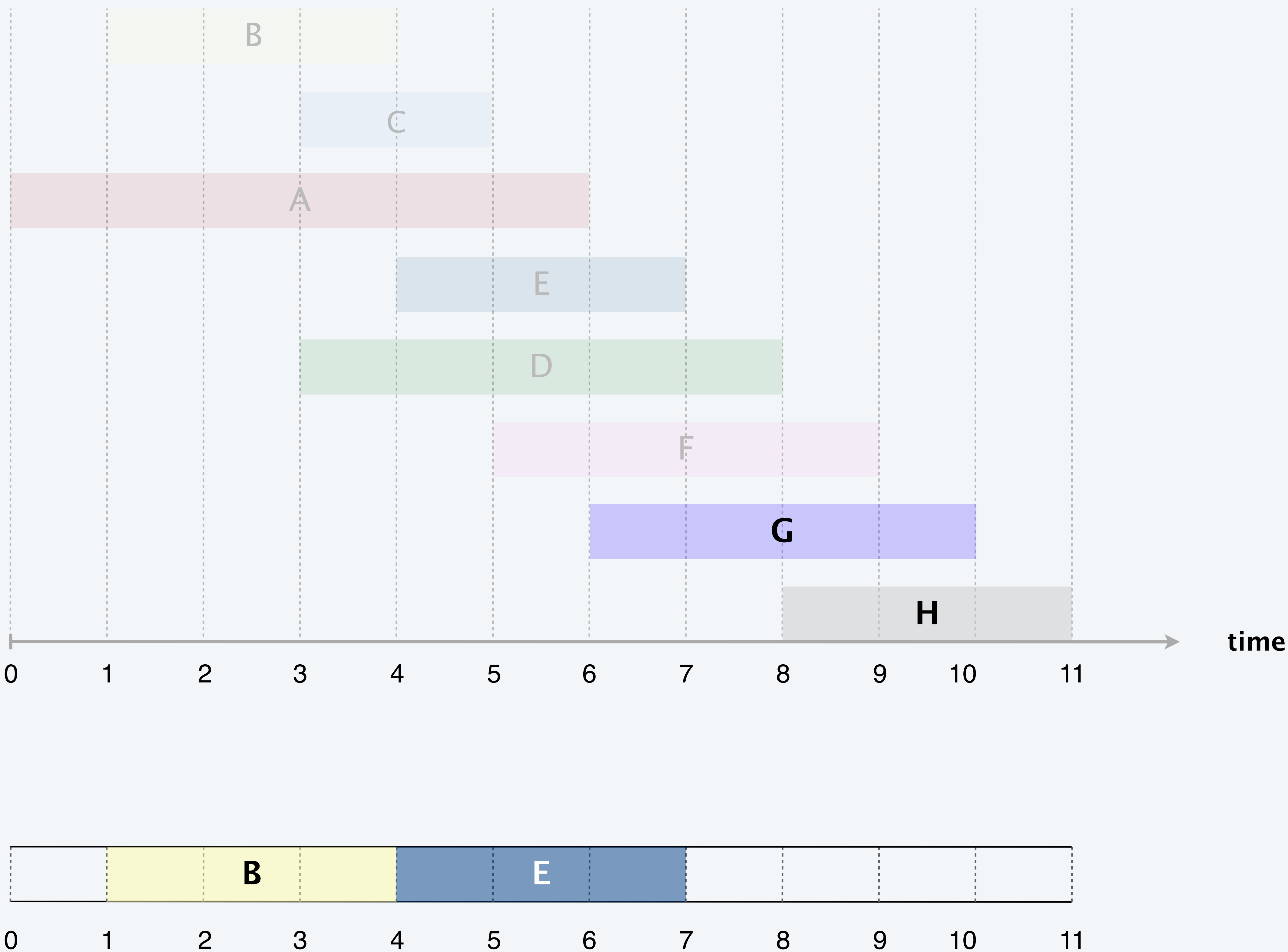
Earliest-finish-time-first algorithm demo



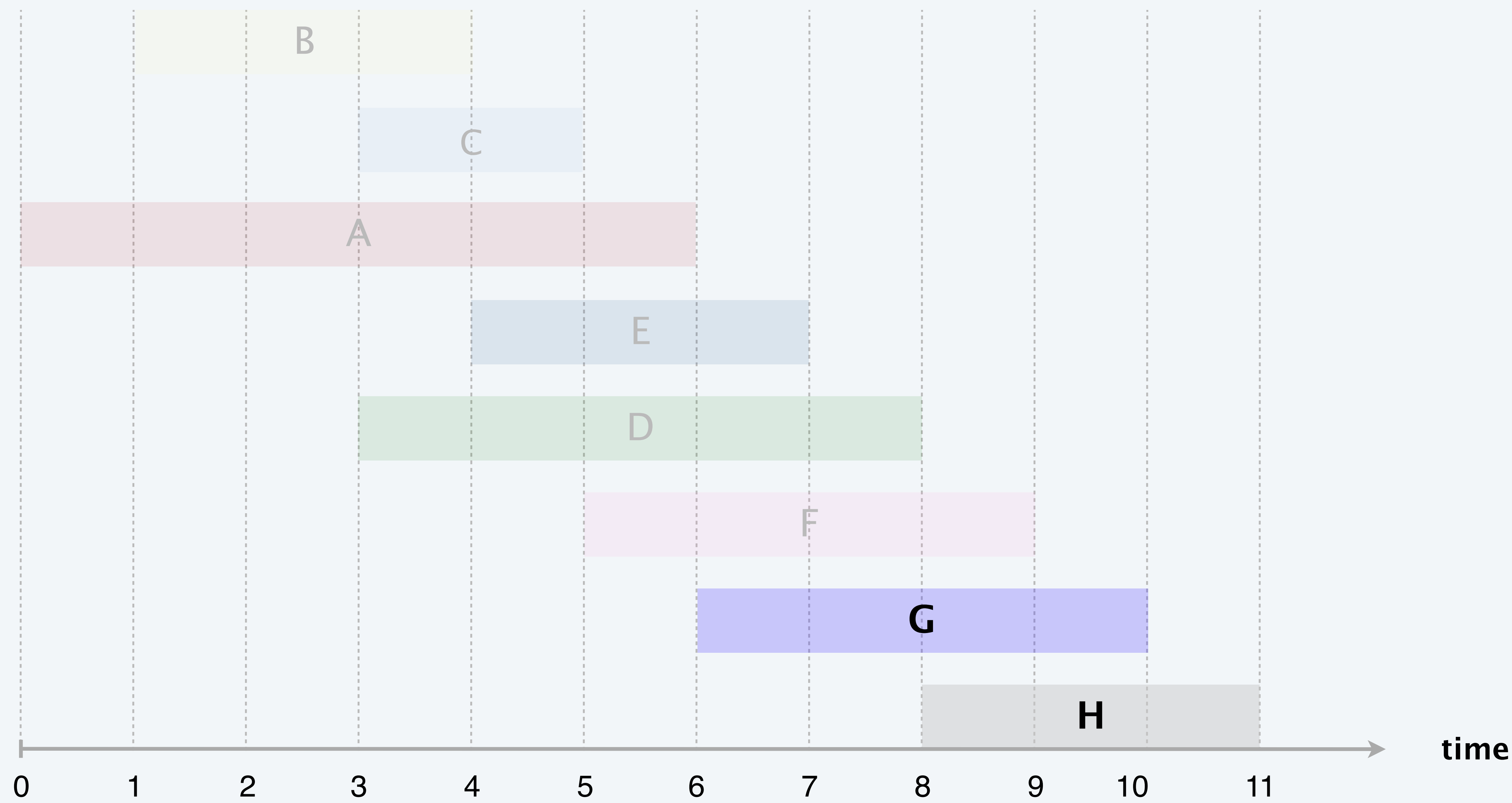
job F is incompatible (do not add to schedule)



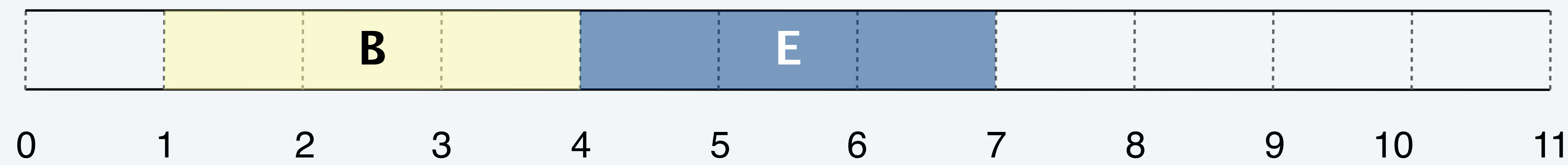
Earliest-finish-time-first algorithm demo



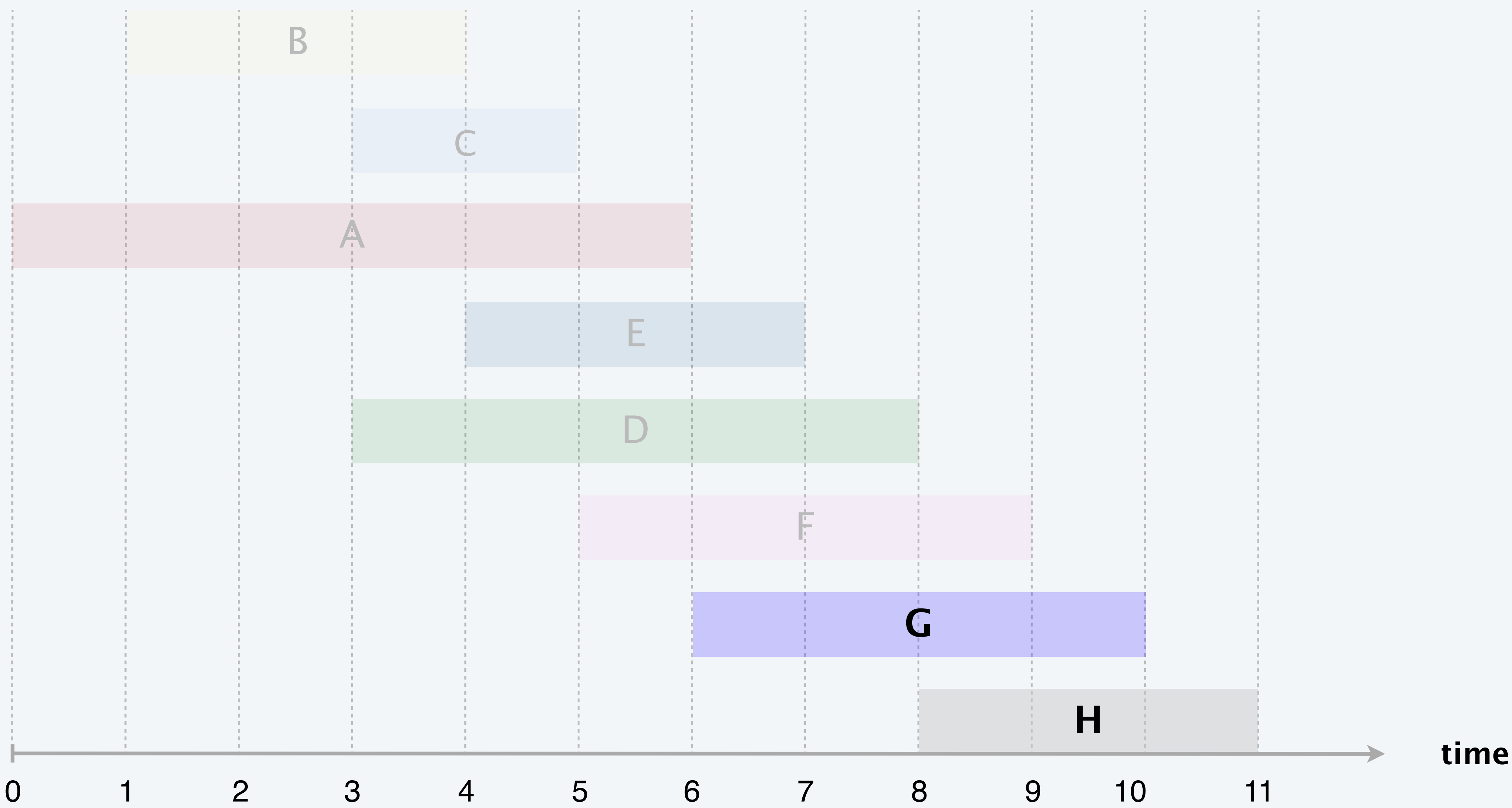
Earliest-finish-time-first algorithm demo



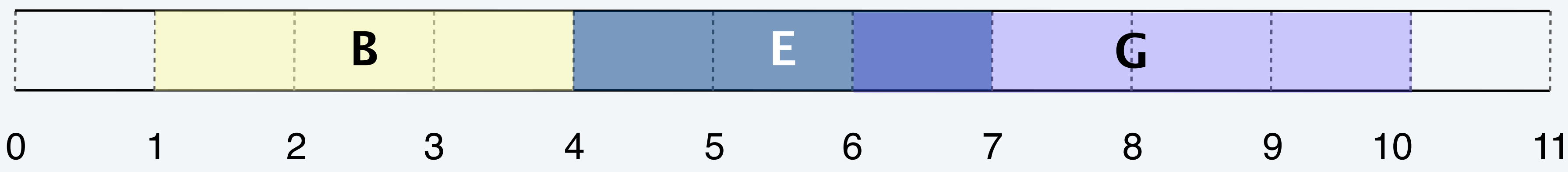
job G is incompatible (do not add to schedule)



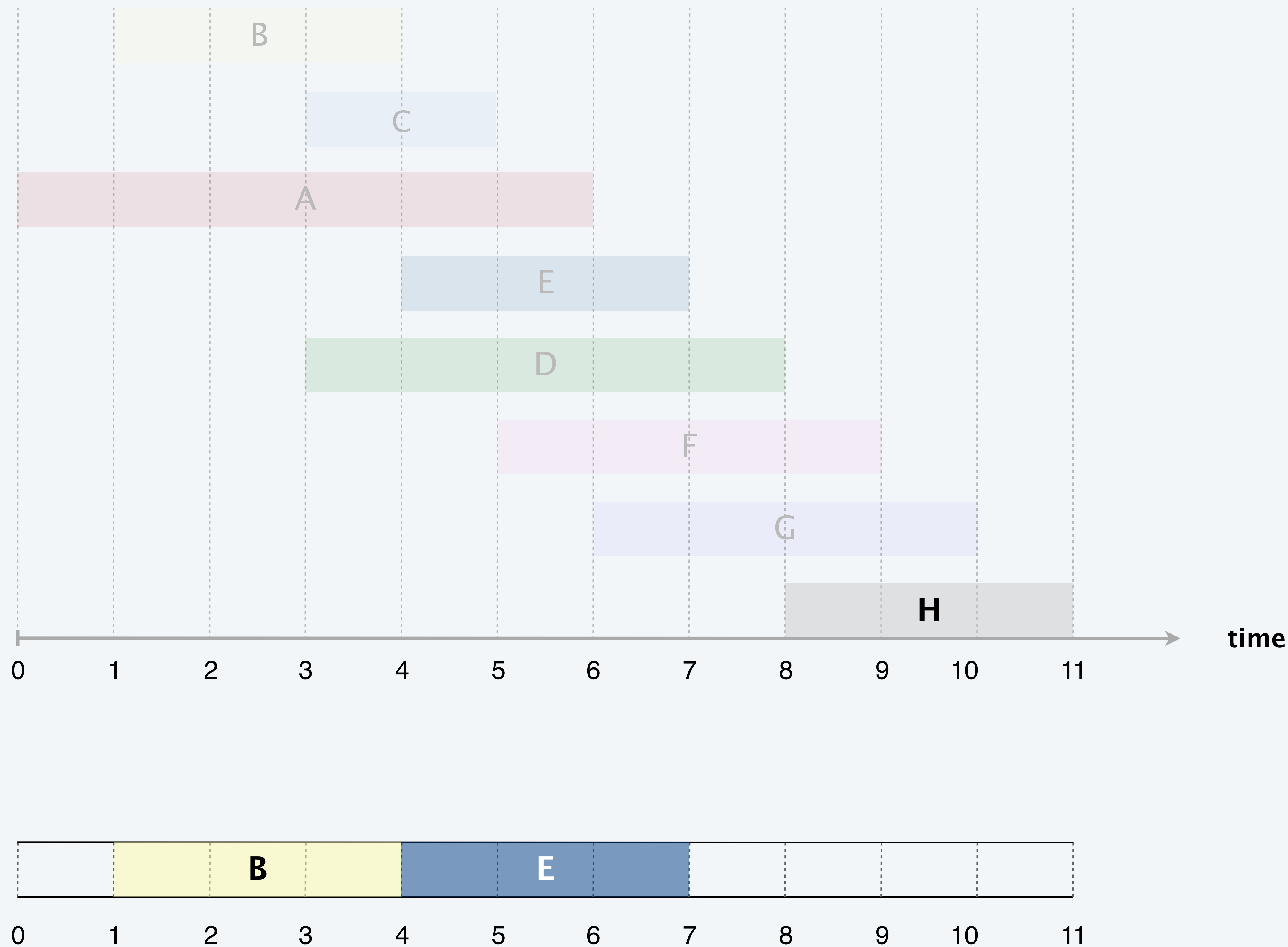
Earliest-finish-time-first algorithm demo



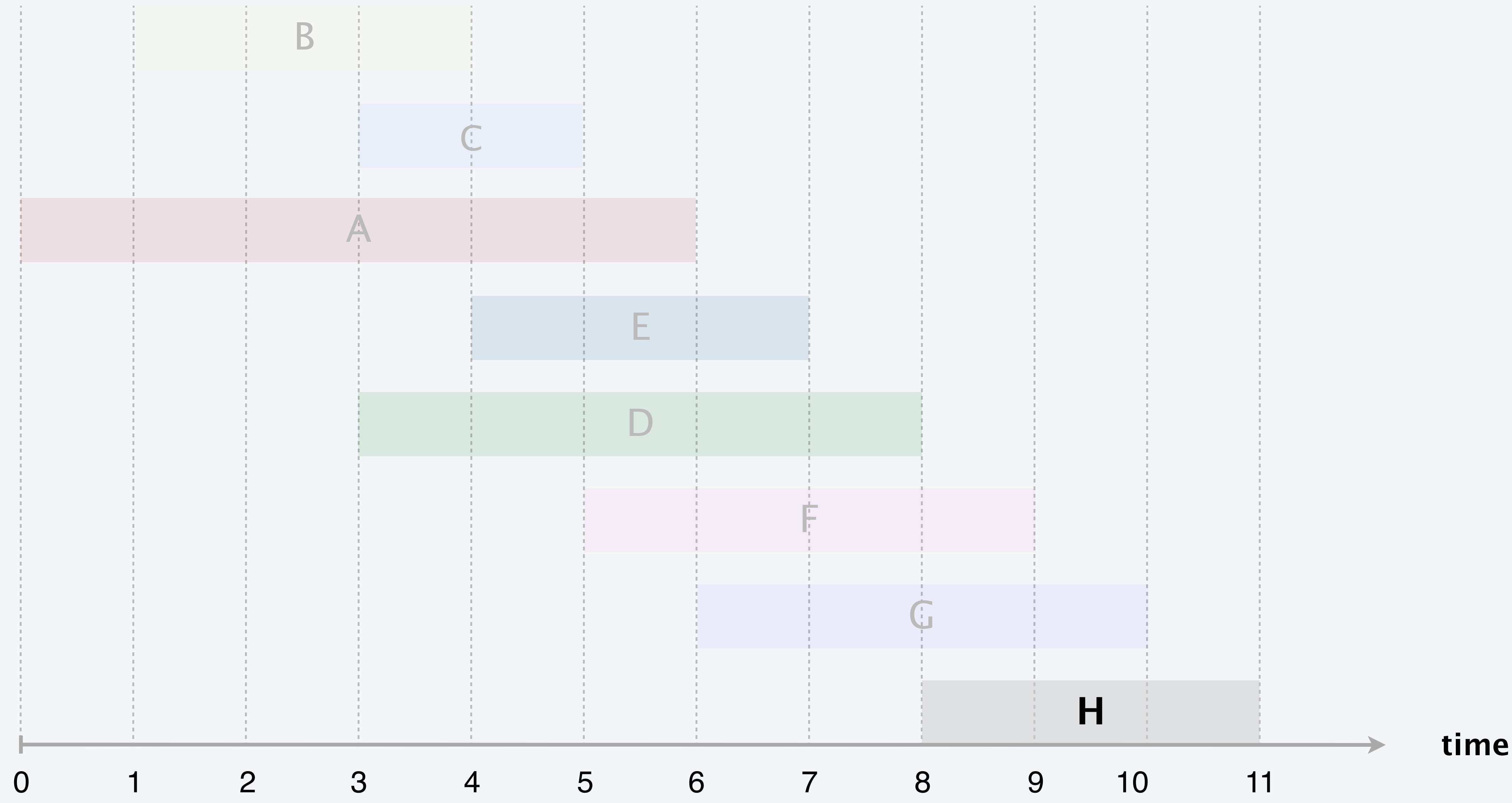
job G is incompatible (do not add to schedule)



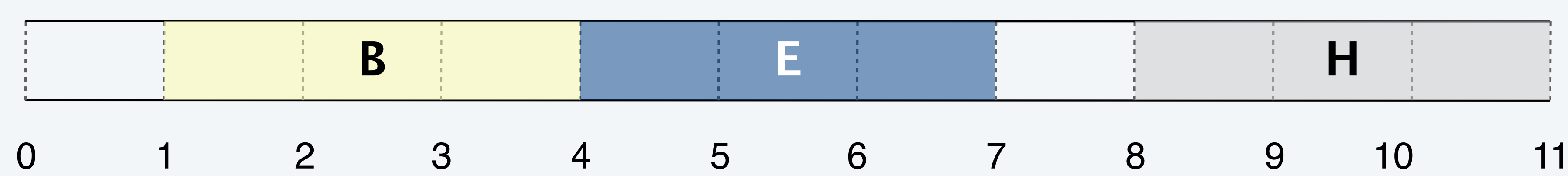
Earliest-finish-time-first algorithm demo



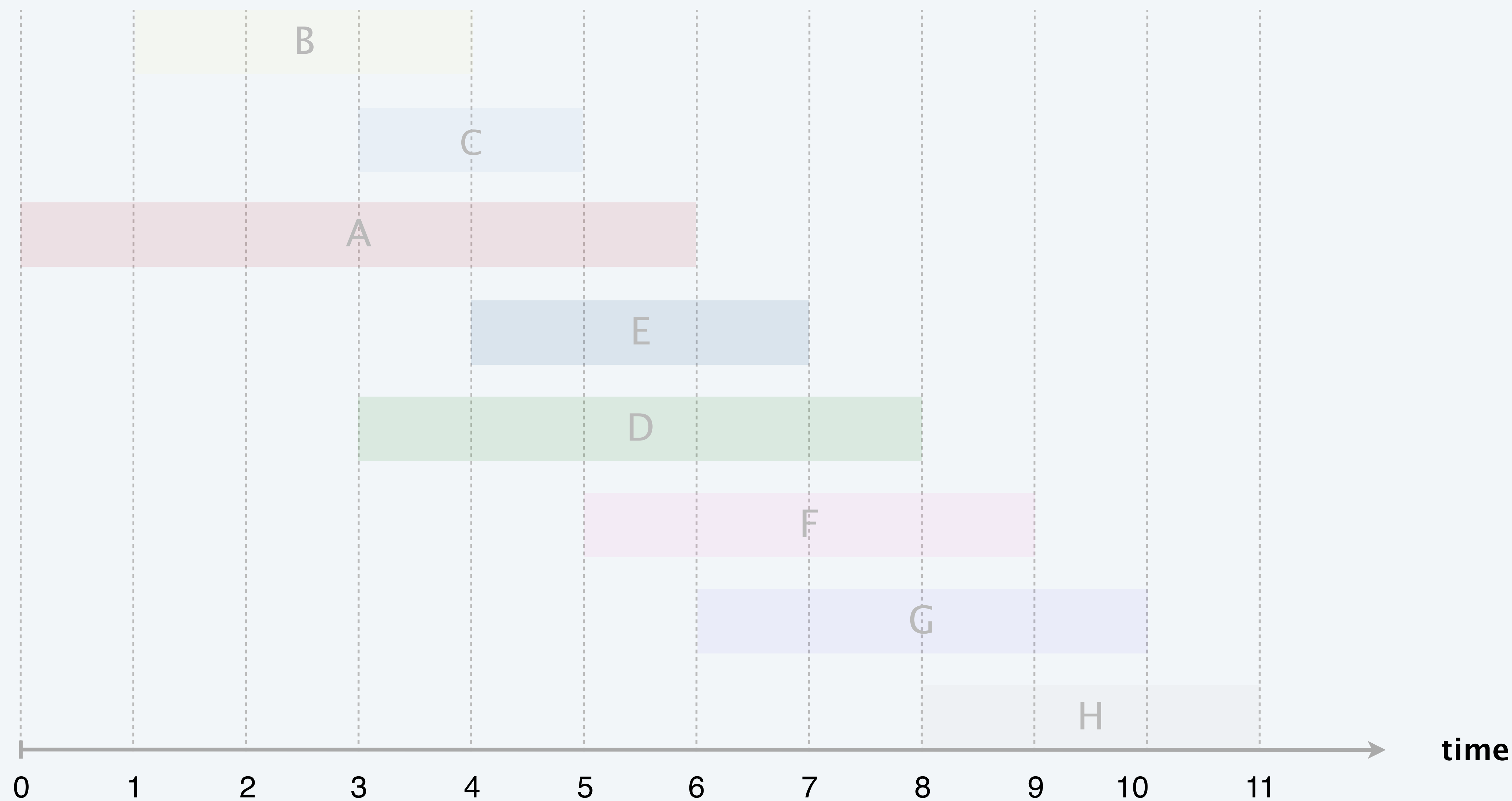
Earliest-finish-time-first algorithm demo

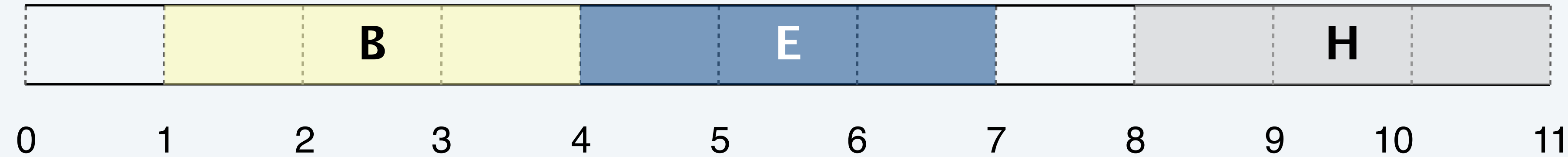


job H is compatible (add to schedule)

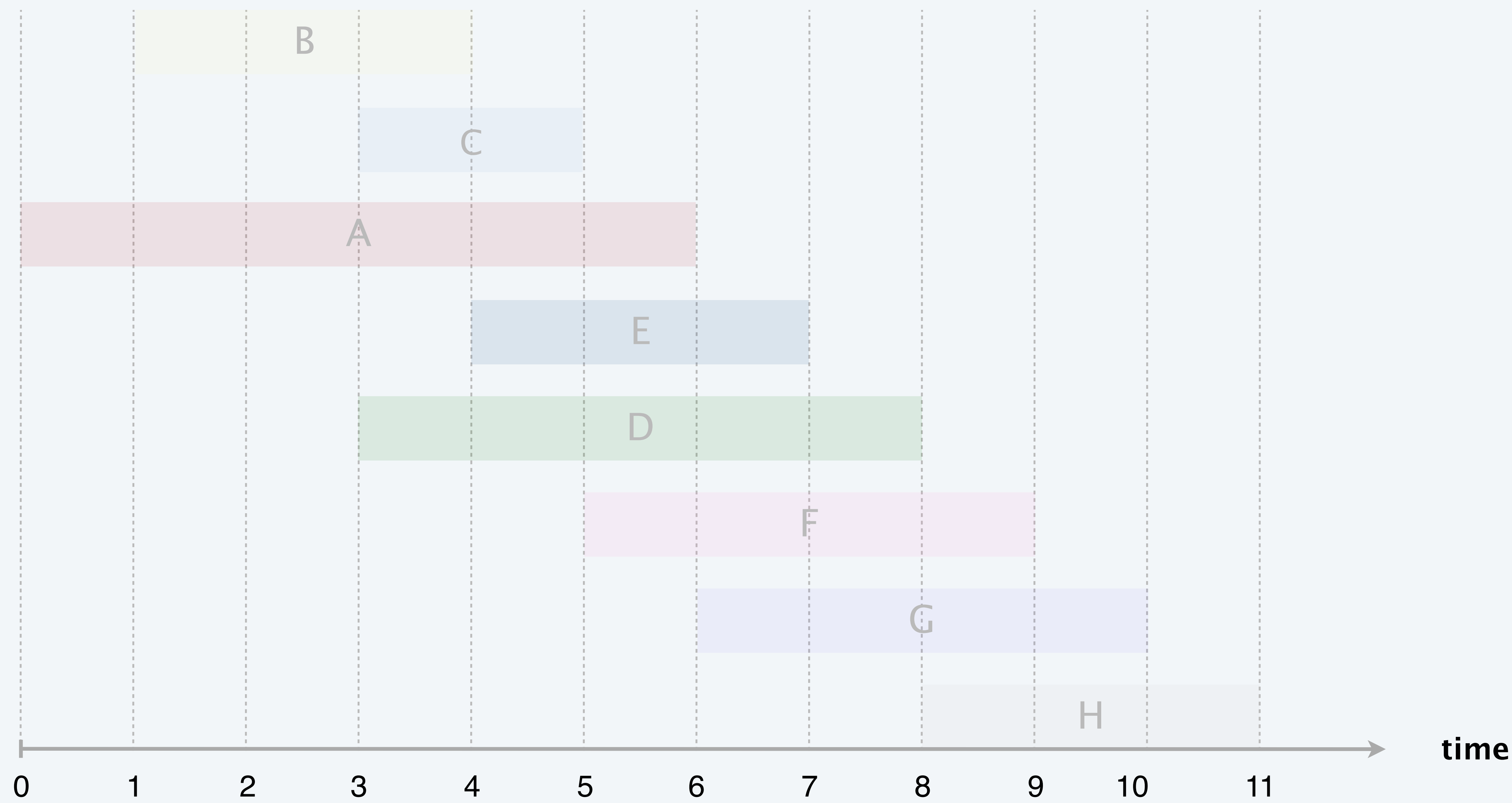


Earliest-finish-time-first algorithm demo

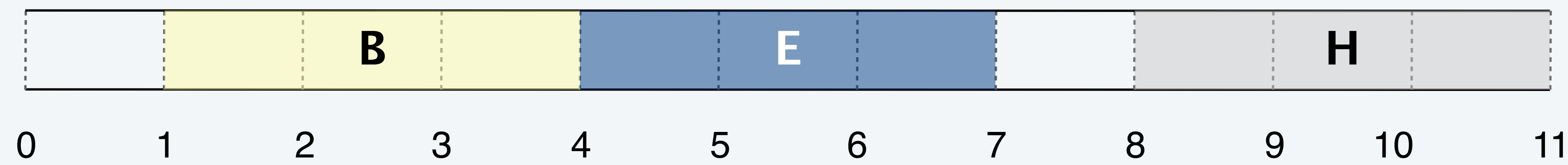




Earliest-finish-time-first algorithm demo



done (an optimal set of jobs)



Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

jobs

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Proof of correctness

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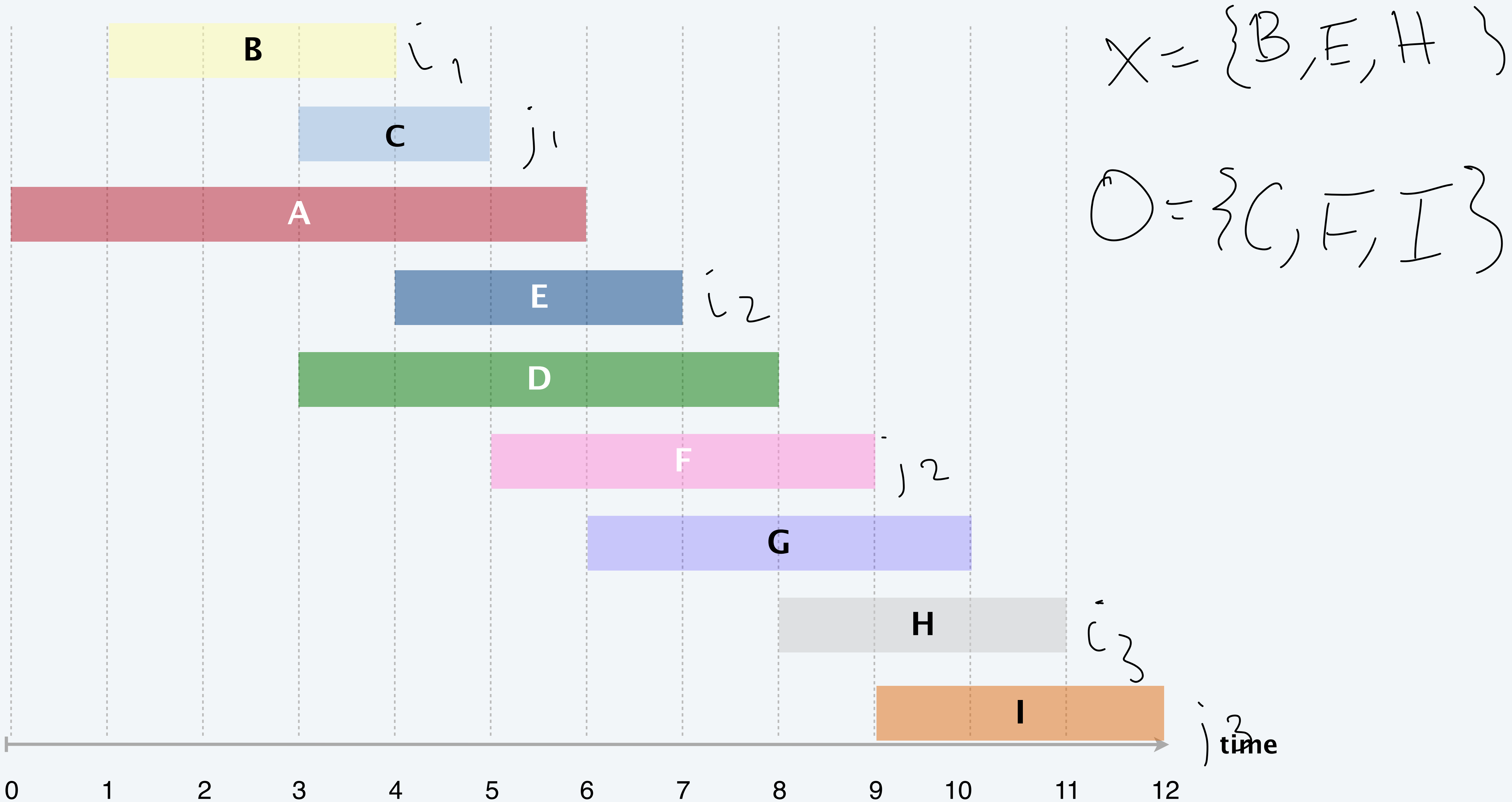

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Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Proof of correctness

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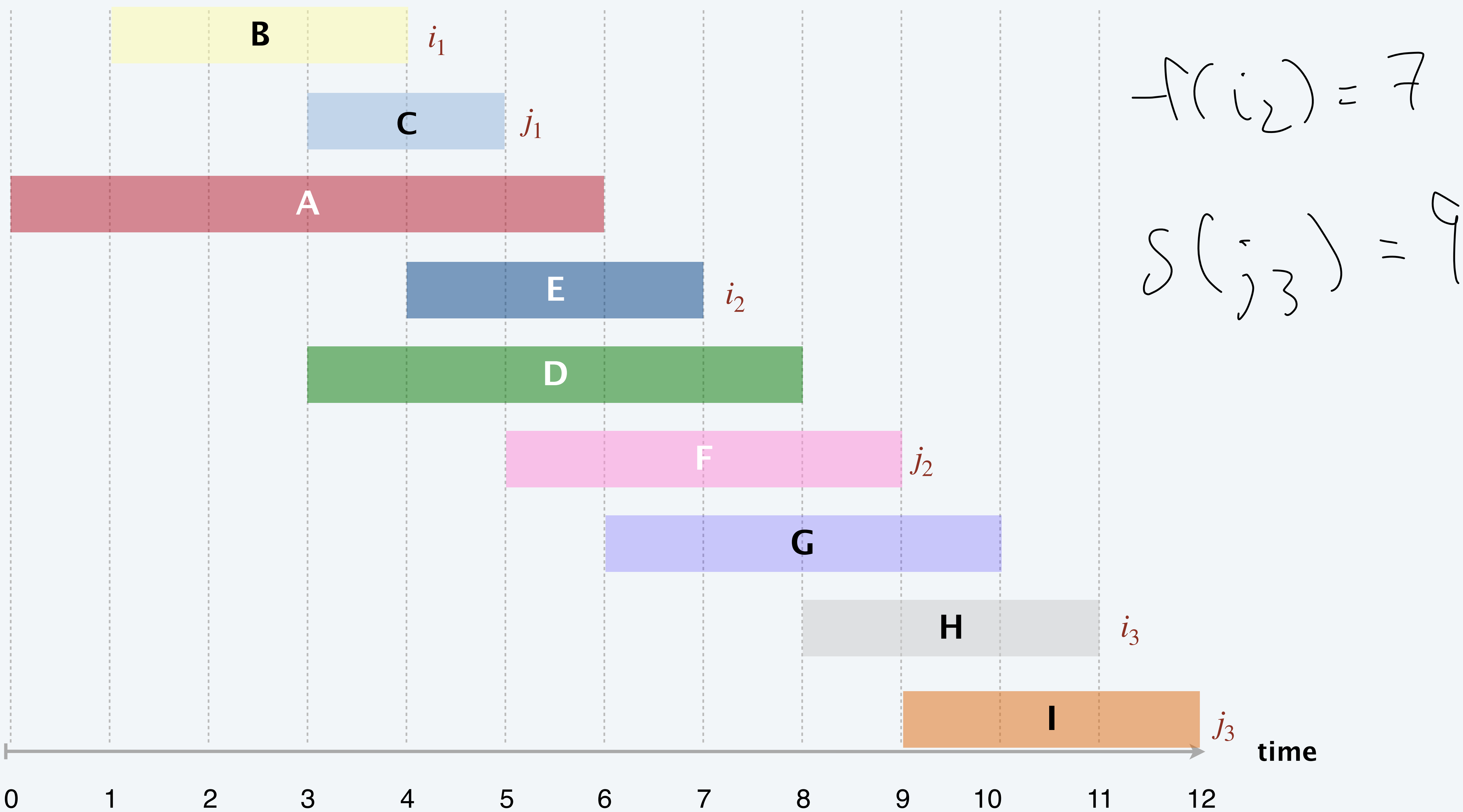

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Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

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Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

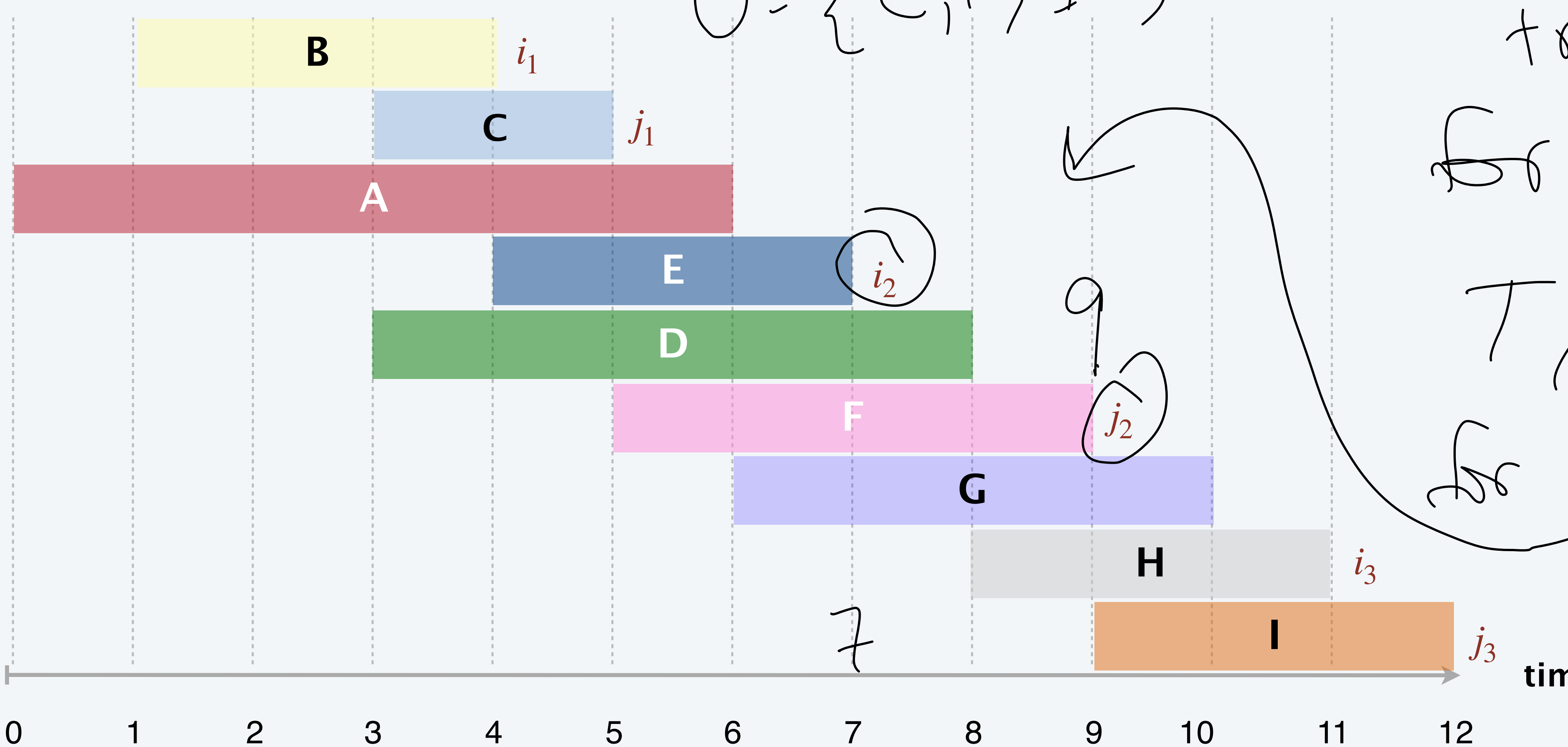
Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

$f(i_2) \leq f(j_2)$

$X = \{B, E, H\}$
$O = \{C, F, I\}$

is it true for $r = 2$?

T/F



Proof of correctness

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Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Assume arbitrary $r \leq k$.

Inductive hyp

Base case

Inductive

For r, $f(i_r) \leq f(j_r)$, and because r was arbitrary, the claim holds for all $r \leq k$.

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof: let $r \leq k$. assume arbitrary instance.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$. inductive hypothesis.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first. base case.

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof: let $r \leq k$.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

inductive case

If $r \neq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$.

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
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Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof: let $r \leq k$.

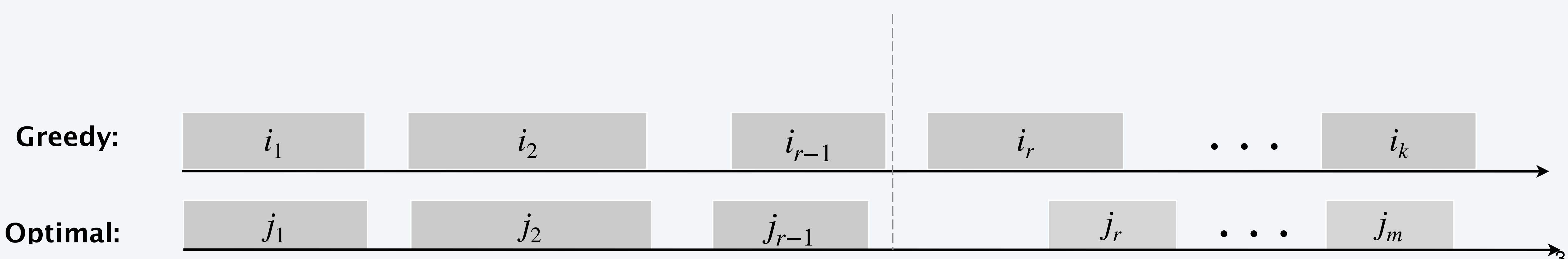
Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

$f(i_r) \leq f(j_r)$

If $r \geq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$.



Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
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Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

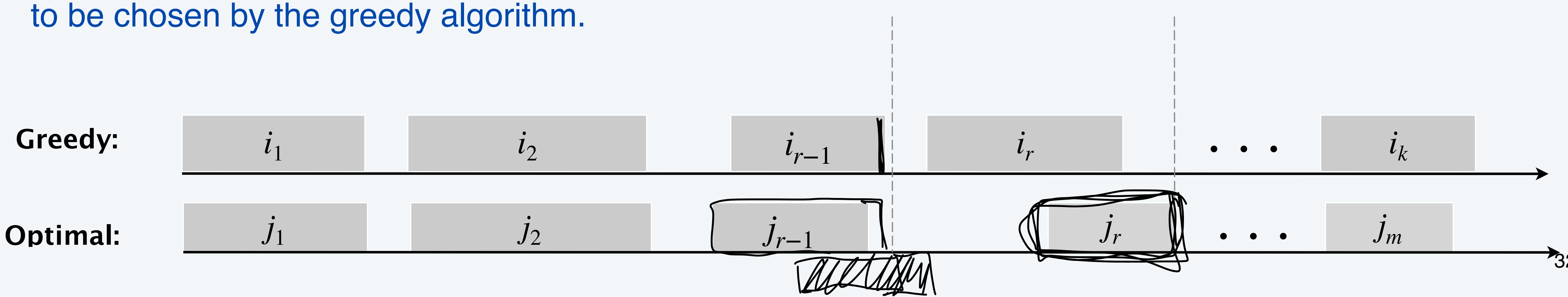
Proof: let $r \leq k$.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

If $r \geq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$. Notice that $f(j_{r-1}) \leq s(j_r)$, so j_r must be available to be chosen by the greedy algorithm.



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Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

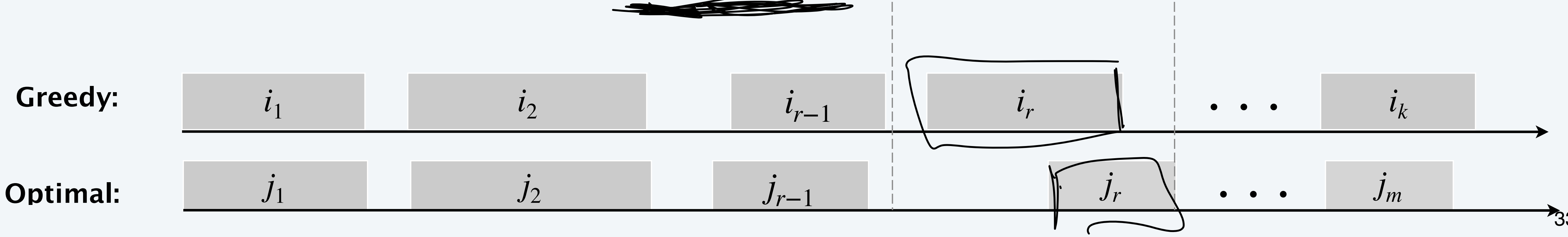
Proof: let $r \leq k$.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

If $r \geq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$. Notice that $f(j_{r-1}) \leq s(j_r)$, so j_r must be available to be chosen by the greedy algorithm. So $f(i_r) \leq f(j_r)$.



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Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof: let $r \leq k$.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

If $r \geq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$. Notice that $f(j_{r-1}) \leq s(j_r)$, so j_r must be available to be chosen by the greedy algorithm. So $f(i_r) \leq f(j_r)$.

Because the claim is true in all cases, it holds.

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

Let $f(i)$ be the finishing time of job i and $s(i)$ be the starting time of job i.

Claim: for all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$.

Proof: let $r \leq k$.

Assume that for all $\ell < r$, we have $f(i_\ell) \leq f(j_\ell)$.

There are two cases:

If $r = 1$, we know that $f(i_1) \leq f(j_1)$ because the greedy algorithm chooses the job with earliest finishing time first.

If $r \geq 1$, we know by the IH that $f(i_{r-1}) \leq f(j_{r-1})$. Notice that $f(j_{r-1}) \leq s(j_r)$, so j_r must be available to be chosen by the greedy algorithm. So $f(i_r) \leq f(j_r)$.

Because the claim is true in all cases, it holds.

Have we shown our original claim yet?

Proof of correctness

Let O be an optimal set of intervals and X be the set of intervals that our algorithm chooses. We want to show that $|X| = |O|$.

greedy

Let $i_1, i_2, \ldots, i_k$ be the set of requests in the order they were added to X. Note that $|X| = k$.
Let $j_1, j_2, \ldots, j_m$ be the set of requests in order of start/finish time in O. Note that $|O| = m$.

For all indices $r \leq k$, we have $f(i_r) \leq f(j_r)$. — "greedy alg stays ahead"

Claim: k=m.

Proof:
Suppose, for the sake of contradiction, that $m \neq k$. That is, X and O have different numbers of jobs, and since O is optimal, $m > k$. Applying the above theorem with $r = k$, we have $f(i_k) \leq f(j_k)$. Since $m > k$, there must be a job in O called j_{k+1}. This job starts after job j_k ends, so job j_{k+1} is compatible w/ X. But that's a contradiction, so $m = k$.

In summary:

- proved greedy alg stayed ahead
- this implies that greedy is opt

a word problem for you (handout)

back at 10:20