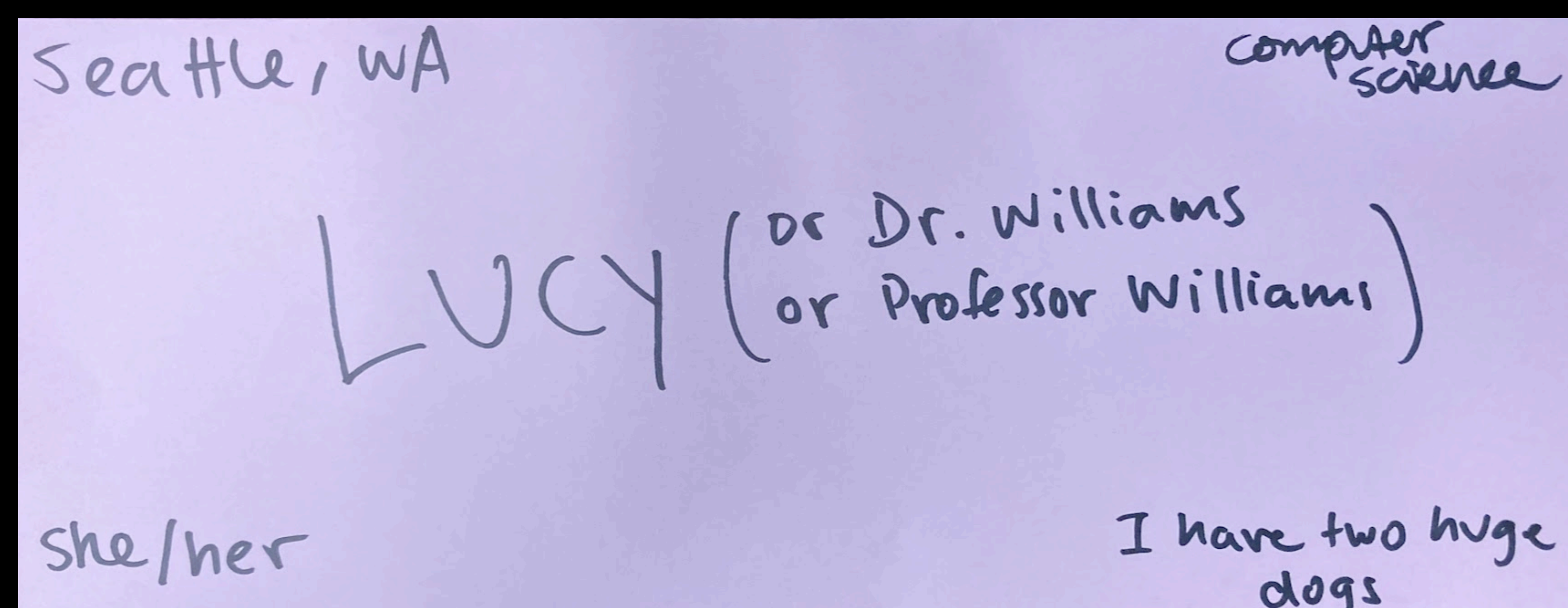
CSCI 332: Advanced Algorithms & Data Structures

Instructor: Lucia Williams

After you sit down, please fold your paper hot dog style and write:

- What you'd like to be called
- Your hometown
- Your pronouns
- Your major/concentration
- A fun fact about you



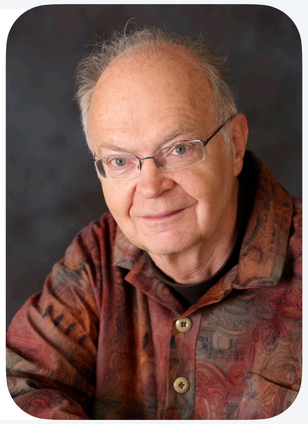
Introduce yourself to your neighbors!

Algorithm definition

Algorithm definition

“ An algorithm is a finite, definite, effective procedure, with some input and some output. ”

— Donald Knuth

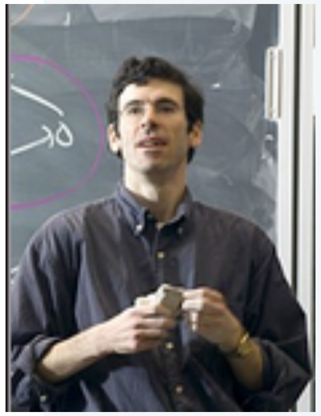


But…

But...

"Algorithmic problems form the heart of computer science, but they rarely arrive as cleanly packaged, mathematically precise questions. Rather, they tend to come bundled together with lots of messy, application-specific detail, some of it essential, some of it extraneous."

— Kleinberg & Tardos



CSCI 232 vs. CSCI 332

What were the focuses of CSCI 232?

CSCI 232 vs. CSCI 332

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CSCI 232. Implementation and consumption of classic algorithms.

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- Fundamental data structures (arrays, stacks, queues, etc.).

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- Searching.

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- Searching.
- Graph algorithms.

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- Graph algorithms.
- String processing.

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

- Fundamental data structures (arrays, stacks, queues, etc.).
- Sorting.
- Searching.
- Graph algorithms.
- String processing.
- Compression.

```
private static void sort(double[] a, int lo, int hi)  {
  if (hi <= lo) return;
  int lt = lo, gt = hi;
  int i = lo;
  while (i <= gt) {
    if      (a[i] < a[lo]) swap(a, lt++, i++);
    else if (a[i] > a[lo]) swap(a, i, gt--);
    else                   i++;
  }

  sort(a, lo, lt - 1);
  sort(a, gt + 1, hi);
}
```

Emphasizes critical thinking, problem-solving, and code.

CSCI 232 vs. CSCI 332

CSCI 332. Design and analysis of algorithms.

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- Finding computational problems in the real world.

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- Greed.

CSCI 232 vs. CSCI 332

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- Divide-and-conquer.

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- Dynamic programming.

CSCI 232 vs. CSCI 332

CSCI 332. Design and analysis of algorithms.

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- Duality.

CSCI 232 vs. CSCI 332

CSCI 332. Design and analysis of algorithms.

- Finding computational problems in the real world.
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- Dynamic programming.
- Duality.
- Data structures.

CSCI 232 vs. CSCI 332

CSCI 332. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.
- Duality.
- Data structures.
- Intractability.

CSCI 232 vs. CSCI 332

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- Finding computational problems in the real world.
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- Divide-and-conquer.
- Dynamic programming.
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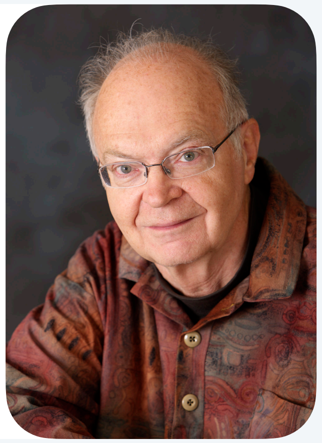
- Intractability.

$$
\begin{aligned}
\sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{2}{j-i-1} &= 2\sum_{i=1}^{n} \sum_{j=2}^{n-i+1} \frac{1}{j} \\
&\leq 2n \sum_{j=1}^{n} \frac{1}{j} \\
&\sim 2n \int_{x=1}^{n} \frac{1}{x} dx \\
&= 2n \ln n
\end{aligned}
$$

Emphasizes critical thinking, problem-solving, and both open-ended problems and rigorous analysis.

Why study algorithms?

" Algorithms are the life-blood of computer science…
the common denominator that underlies and unifies the different branches. " — *Donald Knuth*



Why study algorithms?

Internet. Web search, packet routing, distributed file sharing, ...

Biology. Human genome project, protein folding, …

Computers. Circuit layout, databases, caching, networking, compilers, …

Computer graphics. Movies, video games, virtual reality, …

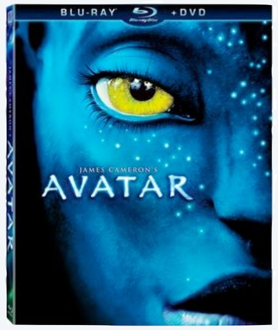
Security. Cell phones, e-commerce, voting machines, …

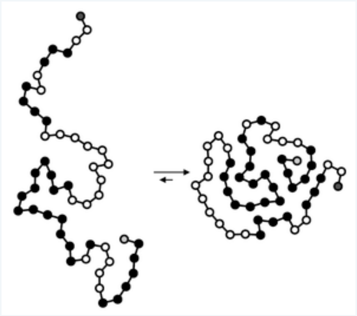
Multimedia. MP3, JPG, DivX, HDTV, face recognition, …

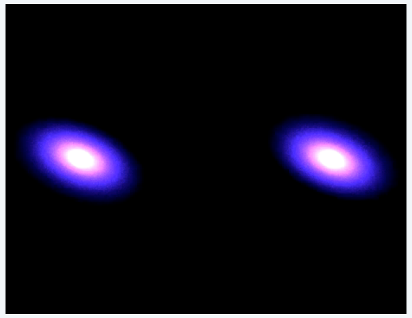
Social networks. Recommendations, news feeds, advertisements, …

Physics. Particle collision simulation, n-body simulation, …

⋮





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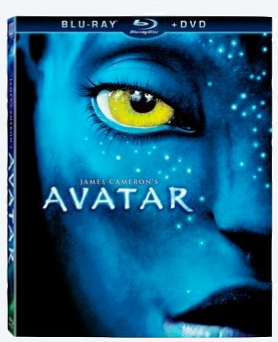


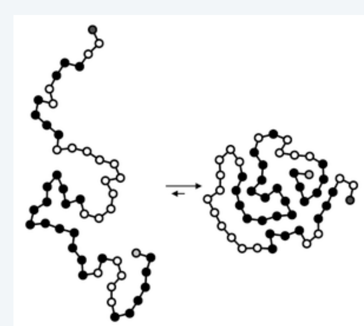

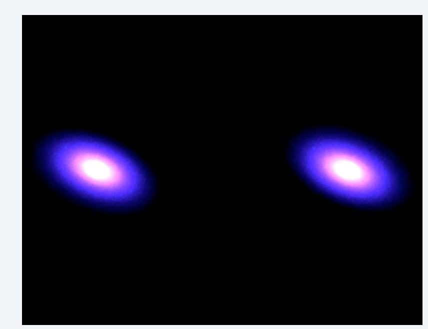

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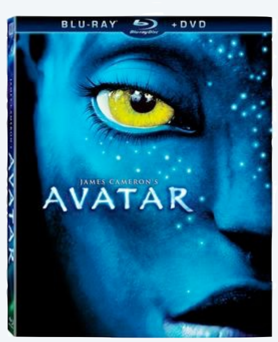


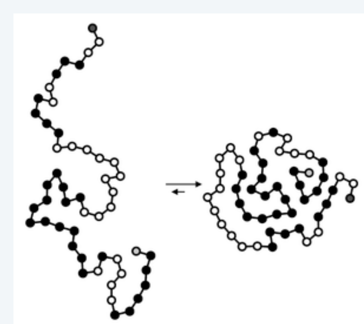

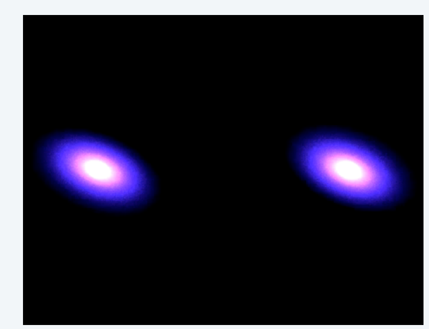

We emphasize algorithms and techniques that are useful in practice.

Course logistics

In table groups, try to complete the syllabus quiz. Some of the questions are open-ended and may not have one single answer!

If your group comes up with a question you can't answer (not necessarily one on the quiz), post it in #questions in Discord.

Matching med-school students to hospitals

How to match? What should we think about when designing an algorithm for this problem?

Matching med-school students to hospitals

Given:

* a set of preferences among hospitals and med-school students

favorite ↓ least favorite ↓

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

favorite ↓ least favorite ↓

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

* a matching of hospitals to students

{ A–Z, B–Y, C–X }

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hospitals' preference lists

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	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

* a matching of hospitals to students

{ A–Z, B–Y, C–X }

With your table group, give at least two *measurable* criterion for a "good" matching.

A common criterion: minimum total score

Given:

* a set of preferences among hospitals and med-school students

favorite → 1st; least favorite → 3rd

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

favorite → 1st; least favorite → 3rd

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

* a matching of hospitals to students

{ A–Z, B–Y, C–X }

3 1

The score is the sum of the ranks for every pair. Smaller scores are better.

Worksheet

You have 15 minutes. Ask for help if needed.

For n hospitals/students, how many unique matchings?

$n!$

Algorithm to finding matching with best score?

brute force – try all

Runtime?

$n!$

$5!$

H_1 $\boxed{5}$

H_2 $\boxed{4}$

H_3 $\boxed{3}$

H_4 $\boxed{2}$

H_5 $\boxed{1}$

– small example

–

Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a self-reinforcing admissions process.

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Unstable pair. Hospital h and student s form an unstable pair if both:

- h prefers s to ~~one~~ of its admitted students.
- s prefers h to assigned hospital.

Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a self-reinforcing admissions process.

Unstable pair. Hospital h and student s form an unstable pair if both:

- h prefers s to one of its admitted students.
- s prefers h to assigned hospital.

Stable assignment. Assignment with no unstable pairs.

- Individual self-interest prevents any hospital–student side deal.

Stable matching problem: input

Input. A set of n hospitals H and a set of n students S.

one student per hospital (for now)

Stable matching problem: input

Input. A set of n hospitals H and a set of n students S.

- Each hospital $h \in H$ ranks students.

one student per hospital (for now)

favorite

least favorite

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

Stable matching problem: input

Input. A set of n hospitals H and a set of n students S.

- Each hospital $h \in H$ ranks students.
- Each student $s \in S$ ranks hospitals.

one student per hospital (for now)

favorite | least favorite

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

favorite | least favorite

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

Stable matching problem: output

Def. A set $M \subseteq H \times S$ is a matching if and only if:

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a perfect matching M = { A–Z, B–Y, C–X }

Stable matching problem: output

Def. A set $M \subseteq H \times S$ is a matching if and only if:

- Each hospital $h \in H$ appears in at most one pair of M.
- Each student $s \in S$ appears in at most one pair of M.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

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Stable matching problem: output

Def. A set $M \subseteq H \times S$ is a matching if and only if:

- Each hospital $h \in H$ appears in at most one pair of M.
- Each student $s \in S$ appears in at most one pair of M.

Def. A matching M is perfect if $|M| = |H| = |S| = n$.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a perfect matching M = { A–Z, B–Y, C–X }

Unstable pair

Def. Given a perfect matching M, hospital h and student s form an unstable pair if both:

- h prefers s to matched student.
- s prefers h to matched hospital.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
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Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
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A–Y is an unstable pair for matching M = { A–Z, B–Y, C–X }

Unstable pair

Def. Given a perfect matching M, hospital h and student s form an unstable pair if both:

- h prefers s to matched student.
- s prefers h to matched hospital.

Key point. An unstable pair h–s could each improve by joint action.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

A–Y is an unstable pair for matching M = { A–Z, B–Y, C–X }

On your own, think about...

Which pair is unstable in the matching { A–X, B–Z, C–Y } ?

1. A–Y.
2. B–X. unstable
3. ~~B–Z.~~
4. None of the above.

not currently matched
both want to switch

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

On your own, think about...

Which pair is unstable in the matching { A–X, B–Z, C–Y } ?

1. A–Y.
2. B–X.
3. B–Z.
4. None of the above.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

B–X is an unstable pair

Stable matching problem

Def. A stable matching is a perfect matching with no unstable pairs.

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching M = { A–X, B–Y, C–Z }

Stable matching problem

Def. A stable matching is a perfect matching with no unstable pairs.

Stable matching problem. Given the preference lists of n hospitals and n students, find a stable matching (if one exists).

	1st	2nd	3rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1st	2nd	3rd
Xavier	Boston	Atlanta	Chicago
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Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching M = { A–X, B–Y, C–Z }

Do you see any potential issues with using Stable Matching to solve the med student to hospital matching problem?

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

Do stable matchings always exist?

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- $2n$ people; each person ranks others from 1 to $2n-1$.
- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

$A\text{–}B,\ C\text{–}D$

$A\text{–}C,\ B\text{–}D$

$A\text{–}D,\ B\text{–}C$

Do stable matchings always exist?

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- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

A–B, C–D $\Rightarrow$ B–C unstable

A–C, B–D $\Rightarrow$ A–B unstable

A–D, B–C $\Rightarrow$ A–C unstable

Observation. Stable matchings need not exist.

Do stable matchings always exist?

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	1st	2nd	3rd
A	B	C	D
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$A–B$, $C–D$ $\Rightarrow$ $B–C$ unstable

$A–C$, $B–D$ $\Rightarrow$ $A–B$ unstable

$A–D$, $B–C$ $\Rightarrow$ $A–C$ unstable

Observation. Stable matchings need not exist.

What about for our version of stable matching?

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
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	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

A–B, C–D $\Rightarrow$ B–C unstable

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What about for our version of stable matching?

5 minute break

Do stable matchings always exist?

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- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
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D	A	B	C

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$A–C,\ B–D \Rightarrow$ $A–B$ unstable

$A–D,\ B–C \Rightarrow$ $A–C$ unstable

Observation. Stable matchings need not exist.

What about for our version of stable matching?

Let's vote

Do stable matchings always exist?

Stable roommate problem.

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	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
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Observation. Stable matchings need not exist.

What about for our version of stable matching?

5 minute break

Gale–Shapley deferred acceptance algorithm

An intuitive method that guarantees to find a stable matching.

GALE–SHAPLEY (*preference lists for hospitals and students*)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and hasn't proposed to every student)

$s \leftarrow$ first student on h's list to whom h has not yet proposed.

IF (s is unmatched)

Add h–s to matching M.

ELSE IF (s prefers h to current partner h')

Replace h'–s with h–s in matching M.

ELSE

s rejects h.

RETURN stable matching M.

Gale–Shapley demo

initialize M

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

We enter the while loop.

How many valid first steps are there?

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to ????

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Wayne

Wayne accepts (since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Yolanda

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Yolanda

Yolanda accepts (since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Chicago proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

What happens?

Chicago proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Chicago proposes to Wayne

Wayne accepts
(and renounces Atlanta)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Val

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Val

Val accepts
(since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Val

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Val

Val rejects
(since she prefers Atlanta)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Yolanda

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Yolanda

**Yolanda accepts
(and renounces Boston)**

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Wayne

Wayne rejects
(since he prefers Chicago)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Val

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Val

Val rejects (since she prefers Atlanta)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Xavier

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Xavier

Xavier accepts
(since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Wayne

Wayne rejects
(since he prefers Chicago)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Yolanda

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Yolanda

Yolanda accepts
(and renounces Detroit)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Xavier

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Xavier

Xavier rejects
(since he prefers Boston)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Wayne

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Wayne

Wayne rejects
(since he prefers Chicago)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Zeus

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Zeus

Zeus accepts
(since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1st	2nd	3rd	4th	5th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1st	2nd	3rd	4th	5th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

STOP
(stable matching)

Can Gale-Shapley ever result in an infinite loop?

1. Yes

2. No

What is the worst-case runtime of Gale-Shapley on an input of size n?

1. $\log n$

2. n

3. n^2

4. $n!$

Proof of correctness: termination

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Pf. Each time through the WHILE loop, a hospital proposes to a new student. Thus, there are at most n^2 possible proposals. ▪

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	1st	2nd	3rd	4th	5th
A	V	W	X	Y	Z
B	W	X	Y	V	Z
C	X	Y	V	W	Z
D	Y	V	W	X	Z
E	V	W	X	Y	Z

	1st	2nd	3rd	4th	5th
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W	C	D	E	A	B
X	D	E	A	B	C
Y	E	A	B	C	D
Z	A	B	C	D	E

n(n–1) + 1 proposals

Does any hospital end up with more than one student?

1. Yes

2. No

Proof of correctness: perfect matching

Claim. Gale–Shapley outputs a matching.

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Pf.

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Claim. In Gale–Shapley matching, all hospitals get matched.

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- But, h proposes to every student, since h ends up unmatched. ※

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Pf. [by counting]

- By previous claim, all n hospitals get matched.
- Thus, all n students get matched. ▪

Proof of correctness: perfect matching

Claim. Gale–Shapley outputs a matching.

Proof of correctness: stability

both want to switch

Claim. In Gale–Shapley matching M^*, there are no unstable pairs.

Let (h, s) be an unmatched pair.

case 1: h never proposed to s.

h's current match is better than s.

h-s not unstable

case 2: h proposed to s.

s rejected h.

s got a better option. h-s is not unstable.

(h, s) is not an unstable pair.

And so, in M^*, there are no unstable pairs. □

Proof of correctness: stability

Claim. In Gale–Shapley matching M^*, there are no unstable pairs.

Pf. Consider any pair h–s that is not in M^*.

$$h - s'$$

$$h' - s$$

$$\vdots$$

Gale–Shapley matching M*

Proof of correctness: stability

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$$\begin{array}{c} h - s' \\ h' - s \\ \vdots \end{array}$$

Gale–Shapley matching M*

Proof of correctness: stability

Claim. In Gale–Shapley matching M^*, there are no unstable pairs.

Pf. Consider any pair h–s that is not in M^*.

- Case 1: h never proposed to s.

  $\Rightarrow$ h prefers its Gale–Shapley partner s' to s.

hospitals propose in decreasing order of preference

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students only trade up

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- In either case, the pair h–s is not unstable. ▪

$h - s'$

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Gale–Shapley matching M*

What have we done?

Using rigorous reasoning, proved useful properties about a real-world problem.

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- Always returns a stable matching (cases)

open questions:

- hospital optimal ?
-

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Why did we focus on stable matching instead of minimum score matching?

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