Name ____

CSCI 332, Fall 2024
Exam 1 (Practice)

Note that this exam has three sections. The first section covers stable matching (40 points), the second section covers analysis of algorithms (40 points), and the third section covers graphs (20 points).

Section 1

Recall the *stable matching problem*: given n men, n women, and preference lists ranking each man for each woman and each woman for each man, find a matching that contains no unstable pairs.

1. An unstable pair satisfies two conditions. What are they?

2. Decide whether the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   True or false? There is an instance of the Stable Matching Problem with a stable matching containing a pair (m, w) such that m is ranked last on the preference list of w and w is ranked last on the preference list of m.

3. Consider the following preference lists.

W: A, B, C, D	A: Y, W, X, Z
X: B, A, D, C	B: W, Y, X, Z
Y: C, D, B, A	C: Y, Z, W, X
Z: D, C, A, B	D: Z, Y, X, W

   What is the outcome of Gale-Shapley on this input?

4. Given the above preference, give another stable matching that is not the output of Gale-Shapley. Here is another copy of the preference lists in case it is helpful.

W: A, B, C, D	A: Y, W, X, Z
X: B, A, D, C	B: W, Y, X, Z
Y: C, D, B, A	C: Y, Z, W, X
Z: D, C, A, B	D: Z, Y, X, W

Section 2

5. Take the following list of functions and arrange them in ascencing order of growth rate. That is, if function $g(n)$ follows function $f(n)$ in your list, then it should be the case that $f(n)$ is $O(g(n))$.

   - $f_1(n) = 10^n$ (10 to the n)
   - $f_2(n) = n^{1/3}$ (n to the one third)
   - $f_3(n) = n^n$ (n to the n)
   - $f_4(n) = \log_2 n$ (log base two of n)
   - $f_5(n) = 2^{\sqrt{\log_2 n}}$ (two to the power of the square root of log base two of n)

6. The *two-sum* problem is as follows. Given an integer t and a sorted array of n integers A, either find the indices of two elements of the array that sum to t or return that no such indices exist.

   Here is an example input to the two-sum problem.

   $A = [-4, 1, 2, 5, 7, 8, 9, 9, 10, 17]$, $t = 11$.

   Give a valid output for the example input.

7. The following algorithm solves the two-sum problem.

```
For i = 1, 2, ..., n
   If there is a j such that A[i] + A[j] = t:
         Return i, j
Return False
```

   Describe a worst-case input for this algorithm.

8. Give an $f(n)$ such that the worst-case runtime of the above algorithm is $\Theta(f(n))$ and explain your reasoning.

9. The following algorithm also solves the two-sum problem.

```
Let i = 1, j = n
While i < j:
  Let s = A[i] + A[j]
  If s equals t:
      Return i, j
  If s < t:
      Let j = j − 1
  If s > t:
      Let i = i + 1
Return False
```

Describe a worst-case input for this algorithm.

10. Give an $f(n)$ such that the worst-case runtime of the above algorithm is $\Theta(f(n))$ and explain your reasoning.

Section 3

11. There's a natural intuition that two nodes that are far apart in a communication network—separated by many hops—have a more tenuous connection than nodes that are closer together. There are a number of algorithmic results that are based to some extent on different ways of making this notion precise. Here's one that involves the susceptibility of paths to the deletion of nodes.

    Suppose that an n-node undirected graph $G = (V, E)$ contains two nodes s and t such that the distance between s and t is strictly greater than $n/2$.

12. Given an example of such a graph with at least 10 nodes.

13. Show that there must exist some node v, not equal to either s or t, such that deleting v from G destroys all s-t paths. In other words, the graph obtained from G by deleting V contains no path from s to t.)

14. Given an algorithm with running time $O(m + n)$ to find such a node v.