10 minutes for quiz

then we will go over it

When you are done w/ quiz, give to Anila and you can do whatever until 10:10

write "I was here" or "I was not here"

re: Wednesday

1. (2 points) For each set notation symbol below, write its name or meaning in English. The first is filled in for you as an example.

(a) $\mathscr{P}$: power set

(b) $\in$: element of, in

(c) $\subseteq$: subset

(d) $\cup$: union

(e) $\cap$: intersection

2. (3 points) Circle True or False for each of the following statements.

(a) $A \cap B = \{x : x \in A \text{ and } x \in B\}$: True or False

(b) $5 \in \{n^2 : n \in \mathbb{Z}\}$: True or False

(c) $\mathscr{P}(S)$ is the set of all elements of S: True or False

$\mathscr{P}(S)$ is the set of all subsets of S

$\mathscr{P}(S) = \{X : X \subseteq S\}$ $S = \{X : X \in S\}$

3. (5 points) Given the following preference lists as input for the hospital/medical student matching problem.

X: A, B, C	A: Y, X, Z
Y: B, A, C	B: X, Y, Z
Z: A, B, C	C: X, Y, Z

(a) Is the matching $M = \{(X,A),(Y,B),(Z,C)\}$ stable? Circle one: Yes or No

(b) How do you know?

no unstable pairs

(c) Is the matching $M = \{(X,C),(Y,B),(Z,A)\}$ stable? Circle one: Yes or No

(d) How do you know?

(X,A) is unstable

GALE–SHAPLEY (*preference lists for hospitals and students*)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and hasn't proposed to every student)

  $s \leftarrow$ first student on h's list to whom h has not yet proposed.

  IF (s is unmatched)

    Add h–s to matching M.

  ELSE IF (s prefers h to current partner h')

    Replace h'–s with h–s in matching M.

  ELSE

    s rejects h.

RETURN stable matching M.

demo

Does G-S always terminate?

Does G-S always return a stable matching?

Which are T about G-S alg?

(1) Once a student becomes matched, they never become unmatched. T

(2) Once a student becomes matched, they only trade up for better hospitals. T

(3) Once a student becomes matched, they only trade down for worse hospitals. F

(4) Once a hospital becomes matched, it never becomes unmatched. F

(5) A hospital only trades up for better students. F

(6) A hospital only trades down for worse students. T

(7) A hospital will never propose to the same student twice. T

(8) The last student a hospital proposes to is the one it ends up with. T

Claim: the while loop executes at most n^2 times.

Proof: Each iteration of the while loop consists of a proposal.

By (7), a hospital will never propose to the same student twice.

Because there are only n^2 pairings of students and hospitals, there are only n^2 possible proposals.

we did not get past here in class

So the while loop executes at most n^2 times.

Claim: The set M returned by G-S is a stable matching.

Proof: Let M be the matching returned by an execution of G-S on some input.

For the sake of contradiction, suppose that there is an unstable pair with respect to M.

This means that there are two pairs $(h,s) \in M$ and $(h',s') \in M$ such that:

- h prefers s' to s

- s' prefers h to h'

so (h,s') is the unstable pair.

By (8), s was the last student h proposed to.

Notice that h prefers s' to s, so by (6) h must have proposed to s' in the past. But by (2), s' only trades up for better hospitals, so s' never would have moved from h to h'.

We have arrived at a contradiction, so no unstable pair can exist. So M is a stable matching.