What did you notice about polynomial functions vs. exponential functions?

When you double the input n to a polynomial function $f = an^b$, the output increases by a constant.

which constant? 2^b

When you double the input n to an exponential function $a b^n$, the output increases by a function of n.

which function of n? b^n

	$f(n) = n^2$	$f(n) = 2^n$	$f(n) = n^{100}$
$n = 10$	100	1028	
$n = 20$	400	$1028 \cdot 1028 \approx 1\text{mil}$	
$n = 100$	10,000	10^{30}	
$n = 200$	40,000	10^{60}	

10^5

10^5

at what point does n^{100} start being better than 2^n?

We say an algorithm is efficient if it is polynomial time.

Is there an efficient alg for stable matching?
Is there an efficient alg for sorting?

Is there a problem for which there is no efficient algorithm?

Why did I have you do the doubling input question?

Can you make any general statements?

cn^a goes up by 2^a when n doubles

kb^n goes up by b^n when n doubles

$$\frac{c(2n)^a}{cn^a} = \frac{2^a n^a}{n^a} = 2^a$$

$$\frac{kb^{2n}}{kb^{2n}} = b^{2n-n} = b^n$$

so one thing we can mean by "scales" is doubling amnt doesn't ↑ as n ↑

what about logs?

$\log_b n$

$$\frac{\log_b 2n}{\log_b n} = \frac{\log_b 2}{\log_b n} + \frac{\log_b n}{\log_b n}$$

$$= \frac{\log_b 2}{\log_b n} + \frac{\log_b n}{\log_b n}$$

$$= 1 + \frac{\log_b 2}{\log_b n}$$

as $n \to \infty$, ?

Asymptotic Bounds

let $f(n)$ and $g(n)$ be functions.

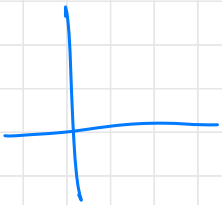
$f(n)$ is $O(g(n))$ if $f(n)$ is asymptotically upper bounded by $g(n)$.

$f(n) = O(g(n))$

more precisely, if $f(n)$ is upper bounded ① by a constant ② multiple of $g(n)$ for sufficiently large ③ n.

ex $f_1(n) = 10n^2 + 3n + 1000$

$g(n) = n^2$ is $f(n) = O(g(n))$?



let's apply def.

① no ← first

②

③ no ← second

ex $f_2(n) = n \log n$

$n(n) = n^4$

is $f(n) = O(g(n))$

$f(n)$ is $\Omega(g(n))$ if $f(n)$ is asymptotically lower bounded by $g(n)$.

fill in w/ groups

more precisely,

w/ groups, is $f_1(n) = \Omega(g(n))$? $f(n) = \Omega(h(n))$?

$f(n)$ is $\Theta(n)$ if $f(n)$ is $O(n)$ and $f(n)$ is $\Omega(n)$

don knuth mnemonics

is