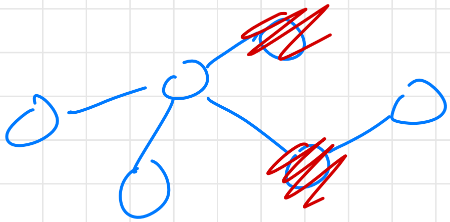


tree

$|V| = 6$

$|E| = 5$

Fill in the following to prove that all trees have one more node than they have edges.

Proof

(1 point) Universal declaration: Let T be an arbitrary tree.

(2 points) Inductive hypothesis (for this proof, use "fewer nodes than" for your definition of "smaller than"):

Assume that for all trees w with fewer nodes than T, w has one more node than it has edges.

There are two cases:

(2 points) ~~Base~~ non-inductive case: Suppose T has one node. (you fill in the rest up to wrap-up sentence)

then T has zero edges.

So T has one more node than it has edges.

(5 points) Inductive case: Suppose T has more than one node. (you fill in the rest. Points divided as follows: 1 point each for create a smaller tree T', applying the inductive hypothesis to T', explaining how to get from T' back to T, and 2 points for explaining how the number of nodes and edges change when going from T' to T.)

Let w be a tree created by removing a leaf node v from T. w has fewer nodes than T, so by IH, w has one more node than it has edges.

when I add v back to w to get T,

nodes: up by 1

edges: up by 1

nodes is 1 more than # edges in T.

Greedy Algorithms

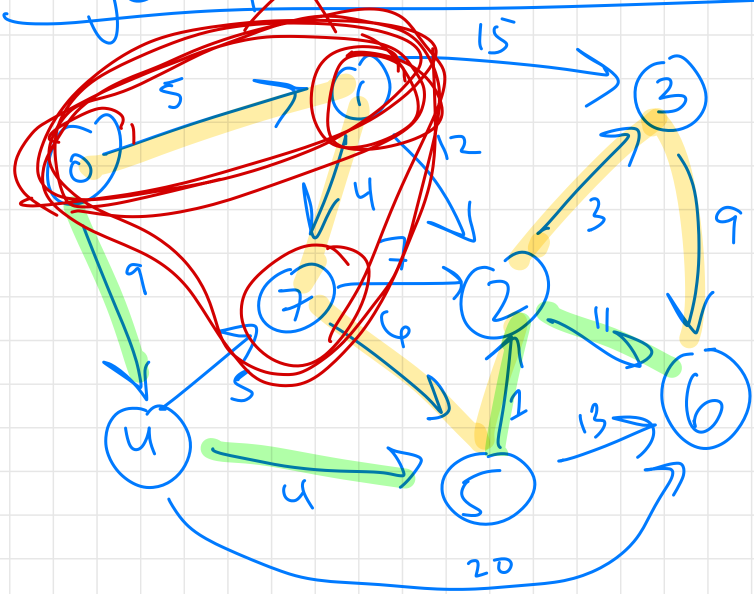
make the best local decision
get an optimal global solution

easy to design, but not always correct

example

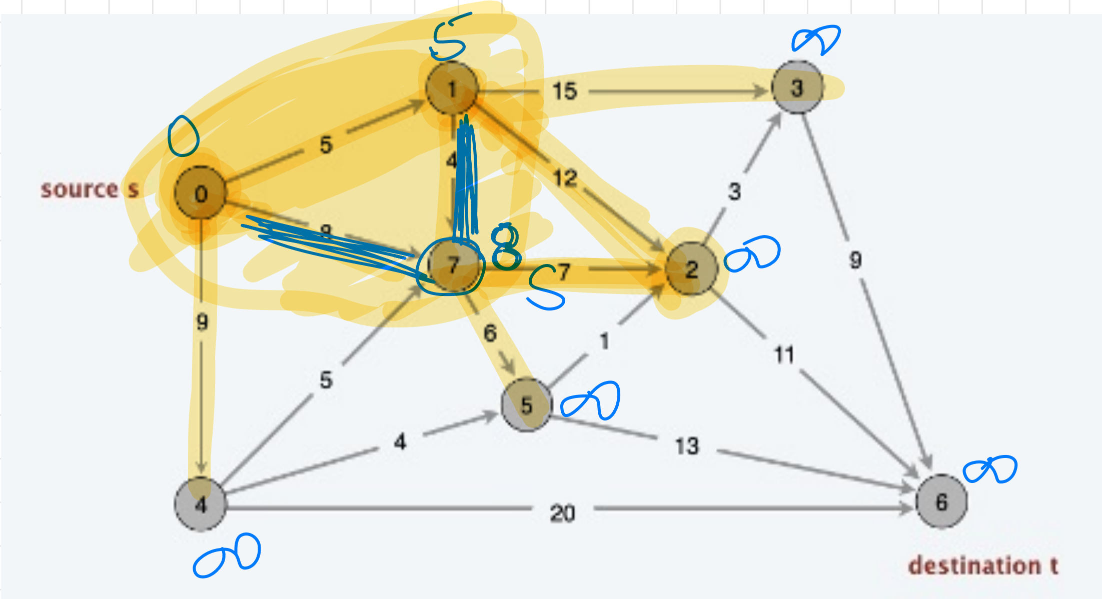
Gale-Shapley
~~Topo Sort~~
~~BFS~~

Single-~~pair~~ source Shortest Paths



$\ell_{(u,v)}$ = length / weight of edge (u,v)

$\ell_{(1,3)} = 15$



$d(0) + l_{(0,v)}$ min ?

for $v = 1$

$d(1) = d(0) + l_{(0,1)} = 0 + 5$

possible v : 4, 7, 2, 3

$d(u) + l_{(u,v)}$

$d(0) + l_{(0,7)} = 0 + 8 = \underline{8}$

$d(1) + l_{(1,7)} = 5 + 4 = 9$

source $s = 0$
target $t = 6$

what is its weight/length?

$9 + 4 + 1 + 11 = 25$

BadGreedy(directed weighted graph G, source s, target t)

start from s

choose shortest edge

keep going until I reach t

27

weighted directed

Dijkstra(G, s, t):

set $d(u)$ for all $u \in V$ to ∞

set $d(s)$ to 0

let S be the set of nodes w/ $d \neq \infty$

while $S \neq V$:

find $v \notin S$ that:

- has an edge out of S
- minimizes $\displaystyle \min_{(u,v):\, u \in S} d(u) + \ell_{(u,v)}$

add v to S, set $d(v)$ to $d(u) + \ell_{(u,v)}$