1. (4 points)

Order the following functions from asymptotically smallest to asymptotically largest. If two functions are asymptotically equal (one is Θ of the other), indicate this with an equals sign. For example, if the functions were n, n^2, and $3n+4$, the answer would be $n = 3n+4 < n^2$.

- 3^n
- 5^n
- $\sqrt{n}$
- $501n^2$
- $\log_2 n$
- $100^{100!}$
- $0.5n^2 - 50n$

$100^{100!} < \log_2 n < \sqrt{n} < 501n^2$

$0.5n^2 - 50n < 3^n < 5^n$

$\Theta(n^2)$ $\quad 3^n \neq \Theta(5^n)$

2. (4 points) For each of the following choices of n_0, c indicate whether they could be used to to prove that $3n^2 + 3$ is $O(n^2)$.

(a) $c = 1, n_0 = 3$. Yes or no?

(b) $c = 4, n_0 = 0$. Yes or no?

(c) $c = 4, n_0 = 10$. Yes or no?

$f \quad g$

$\forall n \geq n_0 : f(n) \leq c\,g(n)$

$c = 4, n_0 = 0$

$n = 0, \quad 3n^2 + 3 \leq 4n^2$

$3 \leq 0$ ✓

3. (3 points) There is an algorithm with best-case runtime that is $\Omega(n^2)$ and worst-case runtime that is $\Omega(n)$. True or false?

✗

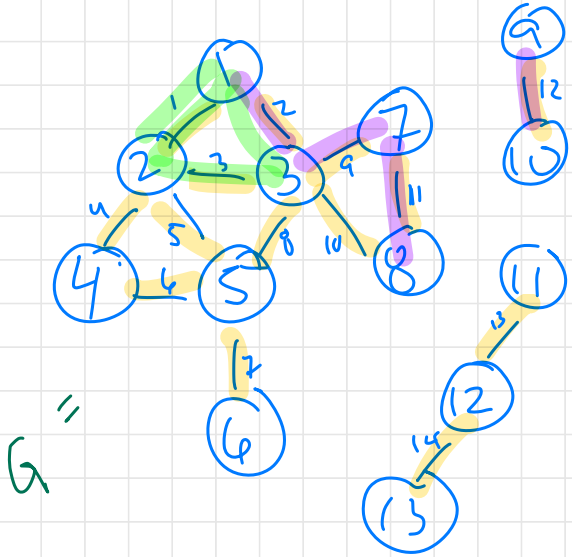
best-case n^2

worst-case n^3

Undirected Graphs

$G = (V, E)$

- V: set of vertices (nodes)
- E: set of edges



$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$

$E = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \ldots\}$

$|V| = 13$

$|E| = 14$

$\{5, 8\} \in E$?

A path in $G = (V, E)$ is a sequence of nodes $v_1, v_2, \ldots, v_k$ with the property that each consecutive pair v_{i-1}, v_i is joined by a different edge.

v_1 v_2 v_3 v_4 v_5 v_6

Is 1, 3, 7, 8, 9, 10 a path?

Is 1, 2, 3, 1, 2 a path?

A simple path has no repeated nodes.

A graph is connected if there is a path between every pair of nodes.

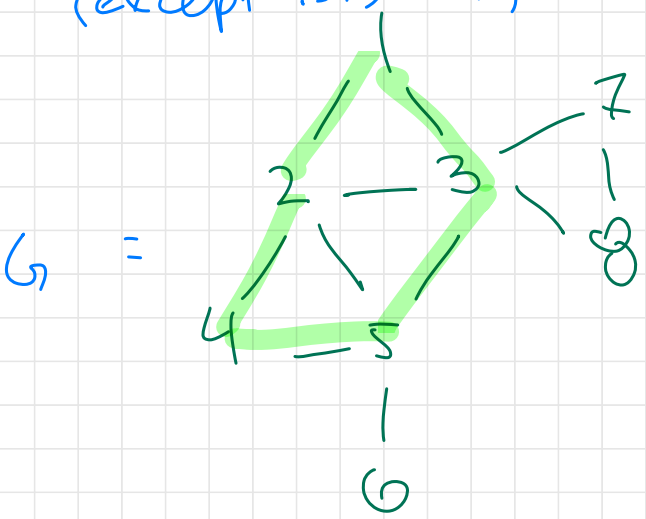
How fast?

$\Omega(n)$

BFS...

A cycle is a path $v_1, v_2, \ldots, v_k$ in which $v_1 = v_k$. and $k \geq 2$

A cycle is simple if all its nodes distinct.
(except 1st, last)



$\{7,7\} \notin E$

$C = 1, 2, 4, 5, 3, 1$ same as $2, 4, 5, 3, 1, 2$

How many simple cycles in G?