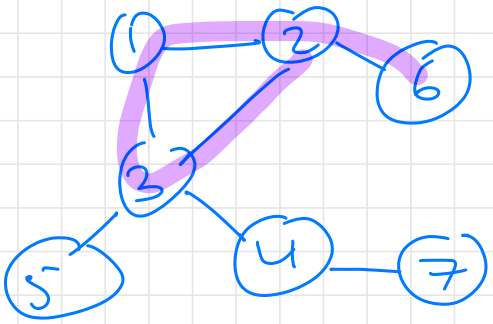
If O in an upper bound (not tight) and can apply to both worst case and best case, why do we say

① "Insertion sort is $O(n^2)$" ? (in the worst case)

Insertion sort is $\Theta(n^2)$ in the worst case

Insertion sort $\Theta(n)$ in best case.

Undirected graphs



$G = (V, E)$

$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

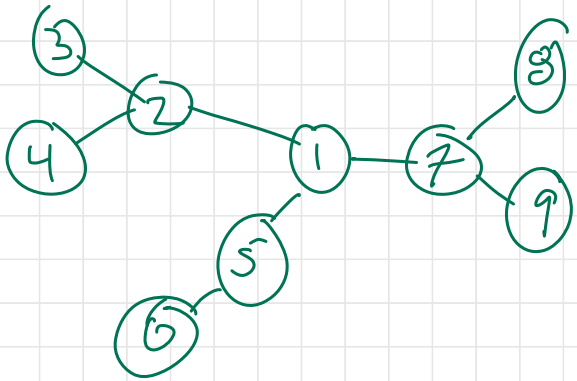
$E = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,5\}, \{3,4\}, \{2,6\}, \{4,7\}, \{8,9\}\}$

(simple) paths

(simple) cycles

connected
$\exists$ path between all pairs of nodes

An undirected graph is a tree if it is connected and it does not contain a cycle.



Claim In a tree, for any pair of nodes u, v, there is a unique path from u to v.

Proof by contradiction boilerplate.

Claim All X have property Y.

All trees have a unique path between any two vertices.

All trees have the property that there is a unique path between all pairs of verts.

Proof Suppose that not all X have property Y.

Then there is an X without property Y.

⋮

Contradiction!

examples:

- actually, that X must have property Y.
- $1=0$

So it must be the case that all X have property Y.

Proof Suppose that not all trees have the property that there is a unique path between all verts.

Let T be such a tree.

Let u, v be the vertices that do not have a unique path in T.

Let P_1 and P_2 be the distinct paths between u and v.

But following P_1 from u to v and P_2 backwards from v to u forms a cycle in T

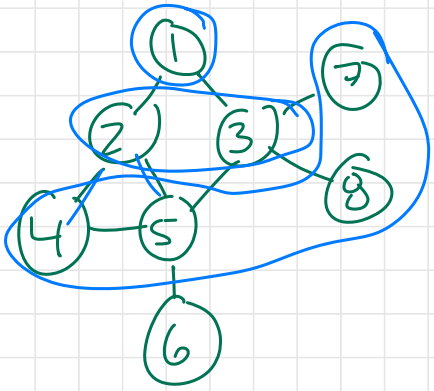
This contradicts that T is a tree.

So in a tree, we must have that all pairs of nodes have a unique path.

Graph Traversal

Depth-first search: dig down as deep as possible

Breadth-first search: flood nodes



$G, s = 1$

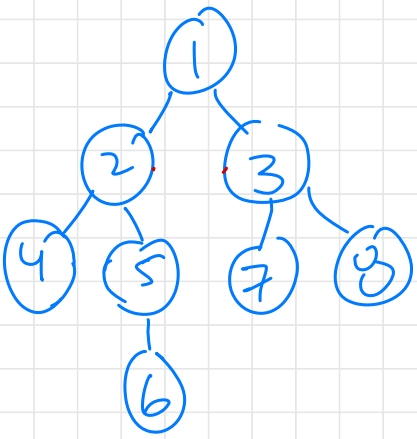
let's think about BFS defining layers.

L_0: starting node

L_1: nodes connected by edges to nodes in L_0

⋮

L_i: nodes connected by edges to nodes in L_{i-1} (and not in any layer yet)



$L_0 = \{1\}$

$L_1 = \{2, 3\}$

$L_2 = \{4, 5, 7, 8\}$

$L_3 = \{6\}$

are layers deterministic?

is the BFS tree deterministic?