

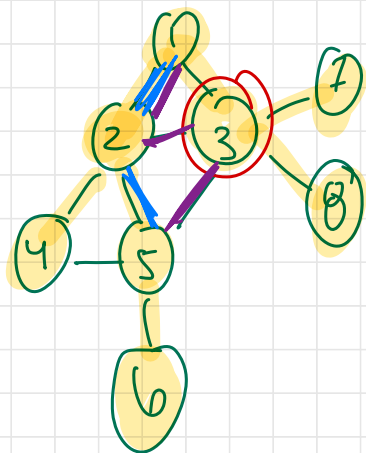
Quiz

BFS trace w/ layers ^{no} PL

Topo sort PL

PF by contradiction PL
some structure given

Graph Traversal

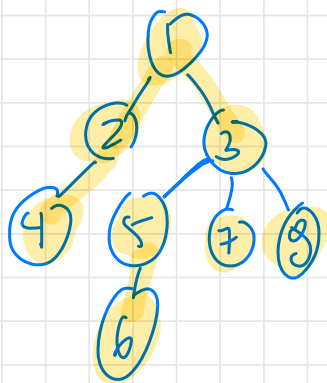


$$G, S = 1$$

$$u = 5$$

$$d(5) = 2$$

BFS Tree



$$L_0 = \{1\}$$

$$L_1 = \{2, 3\}$$

$$L_2 = \{4, 5, 7, 8\}$$

$$L_3 = \{6\}$$

Given G and s , unique layers

BFS tree may not be unique

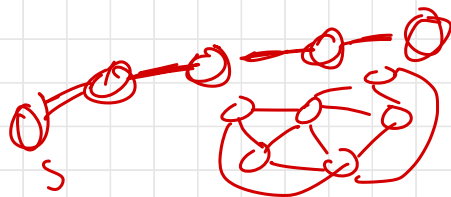
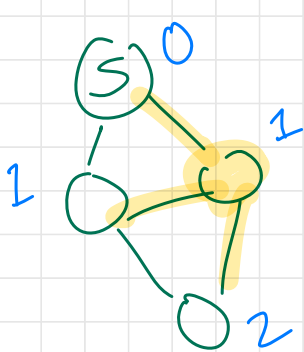
let $d(u)$ be ^{length of} shortest path from s to node u .
How do $d(u)$ and u 's layer L_i relate?

$$d(u) = i$$

BFS (G, s) G connected
 $|V|=n$, $|E|=m$

- ① set all nodes' layer to null ←
set s 's layer to 0
let $L = 0$ ← keep track of current layer

- while there is some node with null layer:
② set layer of all nodes adjacent to a node in L to $L+1$ (if null)
 $L = L+1$



Finds nodes reachable from s .

Runtime? best vs worst

① depends on n Σ, O, Θ ? $\Theta(n)$

② while loop

$O(n)$ # times loop runs

$m+n \in$
 n

$O(n)$ work in loop

m

$n-1$

$O(n^2)$

better

① $\Theta(n) \leftarrow$

② over all while loops:

each node processed once

check all adjacent nodes

$\deg(v)$

$$\sum_{v \in V} \deg(v) = 2m = \Theta(m) \leftarrow$$

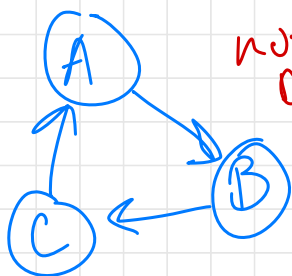
$$\Theta(n+m)$$

$$\text{adj-mtx} \\ \Theta(n^2)$$

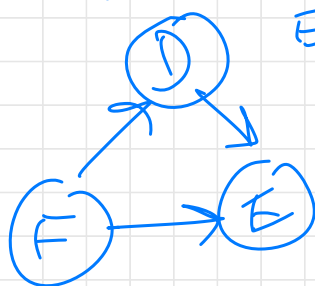
$$\text{adj lists} \\ \Theta(n+m)$$

Directed Acyclic Graphs DAGs

A DAG is a directed graph that contains no directed cycles.



not a
DAG



$$\begin{matrix} \text{edges} \\ \downarrow \\ E = \{(F,D), \\ (D,E), \\ (F,E)\} \end{matrix}$$

A topological order of a directed graph is an ordering of the nodes v_1, v_2, \dots, v_n such that for every edge (v_i, v_j) , $i < j$.

$$\begin{matrix} F, D, E \\ \underline{v_1} \quad \underline{v_2} \quad v_3 \end{matrix}$$

$$\begin{matrix} F \rightarrow D \\ v_1 \quad v_2 \\ D \rightarrow E \\ v_2 \quad v_3 \\ F \rightarrow E \\ v_1 \quad v_3 \end{matrix}$$

$$1 < 2 \quad \checkmark$$

$$2 < 3 \quad \checkmark$$

$$1 < 3 \quad \checkmark$$