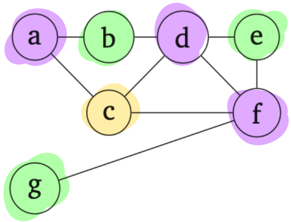
1. (4 points) What are the layers produced by BFS on the following graph starting from node c? If a layer is not present, leave it blank.

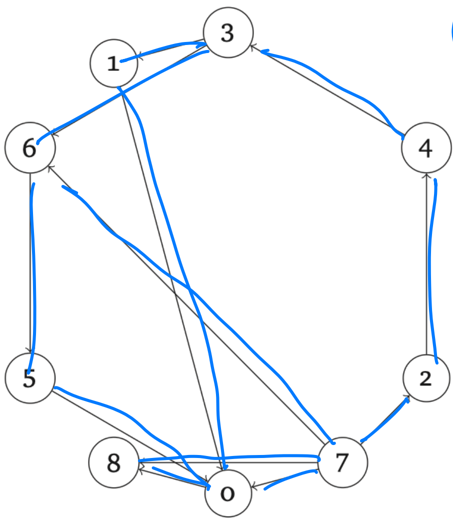


$L_0 = \{$ c $\}$
$L_1 = \{$ a, d, f $\}$
$L_2 = \{$ b, e, g $\}$
$L_3 = \{$ $\}$

2. (3 points) For each of the following orderings of the vertices of the graph below, indicate whether or not it is a valid topological ordering.

$(v_i, v_j) \in E$

$i < j$



bc $1 \to 0$ is an edge

(a) $0, 1, 2, 3, 4, 5, 6, 7, 8$. Yes or no?

(b) $7, 2, 4, 3, 1, 6, 5, 0, 8$. Yes or no?

(c) $7, 2, 4, 3, 6, 5, 1, 0, 8$. Yes or no?

3. (4 points) Order the following statements from 1 to 9 to produce a proof of the following statement: *If G is a DAG, then G has a node with no incoming edges.*

9 Because there is a cycle in G, we have a contradiction with the assumption that G is a DAG.

4 Since v_0 has an entering edge, we can follow it backwards to v_1.

2 Let G be such a DAG where every node has at least one incoming edge.

3 Start at any node v_0 in G.

5 Since v_1 has an entering edge, we can follow it backwards to v_2.

7 Since there are only n nodes, after $n+1$ steps we must have visited a node more than once.

6 Since every node has an entering edge, we can follow this process indefinitely.

1 For the sake of contradiction, assume that not every DAG has a node with no incoming edge.

8 This produces a cycle.

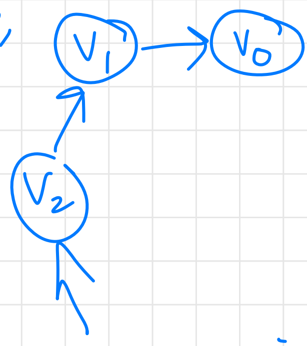
For the sake of contradiction, assume that not every DAG has a node w/ no incoming edges.

Let G be such a DAG where every node has at least one incoming edge.

Start at any node v_0 in G.

Since v_0 has an entering edge, follow backwards to v_1.

Since v_1 has entering edge, follow to v_2



Every DAG has at least one node w/ no incoming edges.

How can you use Claim above to write an alg. for topo sort on a DAG?

toposort (DAG G):

while nodes in G:

find a node w/ no incoming edges

put it in topo order